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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

GENERAL INTRODUCTION

One of the major tasks of time series analysis is to uncover the dynamic behavior of a time series to produce more accurate forecasts. Having access to better forecasts is of practical importance to economic agents and policy makers, e.g., production and financial planning, policy evaluation, and the control and optimization of industrial processes. Therefore, this dissertation focuses on a couple of topics in the area of economic forecasting.

The class of stationary models is the most widely used class of stochastic models for describing time series. However, many time series are nonstationary tending to grow over time, which can be represented as the linear trend model with autoregressive and possibly unit-root errors. Various procedures have been proposed to estimate such model, among which, Falk and Roy (2000) showed that the Prais-Winsten modified weighted symmetric least squares estimation method is the best procedure to use to construct point forecasts for the linear trend model with first-order autoregressive errors. The first chapter of this dissertation is an extension of the paper by Falk and Roy (2000). By using Monte Carlo simulation and bootstrap methods to compare the actual and nominal coverage probabilities of prediction intervals, chapter I examines whether the best point predictor also leads to prediction intervals with the most accurate coverage rates for the linear trend model with first-order autoregressive errors.

Expected rate of inflation is one of the most commonly investigated variables in economics due to its roles played in monetary policy and various economic theories. There are different approaches to measure economic agents' inflation expectations, such as using direct measure from surveys, or derive a proxy from long lags of past prices or inflation rates. An alternative approach, proposed by Hamilton (1992), uses a vector dynamic system to measure expectations about future aggregate prices. The second chapter of this dissertation, chapter II, adopts this innovative methodology to construct inflation expectations by incorporating information in the commodity futures market. The third chapter is closely related to the second chapter by using the constructed time series of inflation expectations to examine two broadly debated topics in the field of economics, the Fisher effect and the Phillips curve. In general, there is controversy among economists over the short-run Fisher effect, but most empirical work tends to support the existence of the long-run Fisher effect. So the short-run Fisher effect is examined in the first part of chapter III. In the second part of chapter III, two main alternative specifications of the Phillips curve, the New Keynesian Phillips curve and the expectations-augmented Phillips curve, are estimated using the GMM method, and the empirical superiority of the two specifications are compared by both the encompassing test and the non-nested test.

CHAPTER I.

PROBABILITY OF COVERAGE INVESTIGATION IN FORECASTING USING THE LINEAR TREND MODEL WITH AUTOREGRESSIVE ERRORS

1. Introduction

One of the major tasks of time-series analysis is to uncover the dynamic behavior of a time series to produce more accurate forecasts. The most widely used class of stochastic models for describing time series is the class of stationary models. However, many time series that arise in economics and industry appear to be non-stationary, tending to grow over time (e.g., nominal and real GDP, the money supply, etc.). One common approach to modeling trending time series is to model the series (or, in some cases, the growth rate of the series) as the sum of a deterministic function of time plus a stationary component.

In this paper, we focus on the problem of forecasting a time series that can be represented as the sum of a deterministic linear function of time and an error process that has a stationary autoregressive representation, in level or first-difference form. This model is called the linear trend model with autoregressive errors. Our attention is limited to the univariate case, which is simple and convenient. Although it may be too simple in many applied settings, studying the simple case often leads to developments that are helpful in the multivariate setting.

We will use Monte Carlo simulation and bootstrap methods to compare the actual and nominal coverage probabilities of (out-of-sample) prediction intervals associated with different estimators of the linear trend model with first-order autoregressive errors, varying the forecast horizon and the autoregressive coefficient (including the unit root case). This paper is an extension of the paper by Falk and Roy (2000), which compares the mean-squared errors corresponding to out-of-sample forecasts produced by different estimators of the linear trend model with autoregressive errors.

The linear trend model with AR(1) errors has the form:

$$Y_t = \alpha + \beta t + u_t = X_t\delta + u_t \qquad t = 1, 2, \ldots, T \tag{1.1}$$

$$u_t = \rho u_{t-1} + \varepsilon_t \qquad \rho \in (-1,1] \tag{1.2}$$

where the ε_t's are independent and identically distributed random variables with zero-mean and constant variance σ^2. The error series is a stationary process if $|\rho| < 1$, and is a random walk process when $\rho = 1$.

Rewriting (1.1) and (1.2) in a more compact representation, we get

$$Y_t = a + bt + \rho Y_{t-1} + \varepsilon_t \tag{1.3}$$

where $a = \alpha(1 - \rho) + \beta\rho$ and $b = \beta(1 - \rho)$.

If the parameters α, β, and ρ (or, equivalently, a, b, and ρ) are known, the h-step-ahead forecast that minimizes the mean squared error conditional upon the lagged Y's is the linear projection of Y_{T+h} on $Y_1, Y_2, \ldots, Y_T$, which can be written as:

$$Y^F_{T+h|T} = \alpha + \beta(T+h) + \rho^h u_T \tag{1.4}$$

where $u_T = Y_T - \alpha - \beta T$.

In reality, the model parameters usually are unknown, so we need to estimate α, β, and ρ as part of the forecasting process. Then the h-step-ahead forecast has the following form:

$$\hat{Y}^F_{T+h|T} = \hat{\alpha} + \hat{\beta}(T+h) + \hat{\rho}^h \hat{u}_T \tag{1.5}$$

where $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\rho}$ are the estimates of α, β, and ρ, respectively. In this case, the forecast errors arise from two sources: the unpredictability of the ε's after the period T; and the errors in estimating α, β, and ρ. When equation (1.4) can be used to forecast the h-step-ahead values, forecast errors arise only from the unpredictability of the ε's following the period T.

In practice, it is important to get accurate estimates for α, β, and ρ to obtain more precise forecasts. We discuss the development of alternative estimators in section 2. Section 2 also introduces the asymptotic prediction variance formula, which can be used to compute the h-step-ahead prediction interval. Section 3 contains the design and results of the Monte Carlo simulation used to compare the coverage properties of prediction intervals constructed from the various estimators. Section 4 presents the design and results of bootstrap method as an alternative way to verify the results we get in the previous session. Section 5 applies the procedures to empirical macroeconomic time series data, and section 6 concludes this paper. An appendix at the end of the paper contains the derivation of the formula used in section 2.

2. Estimators and the Asymptotic Prediction Variance

2.1. Estimators

The most commonly used estimators of the linear trend model are the least squares estimators: ordinary least squares (OLS) and feasible generalized least squares (FGLS). The two OLS estimators are:

OLS_1: Estimate $Y_t = a + bt + \rho Y_{t-1} + \varepsilon_t$ by OLS directly in one step.

OLS_2: Estimate α and β from (1.1) by OLS to obtain $\hat{\alpha}_{OLS}$ and $\hat{\beta}_{OLS}$, compute the residuals $\hat{u}_t = Y_t - \hat{\alpha}_{OLS} - \hat{\beta}_{OLS} t$, and estimate ρ from (1.2) by OLS using the regression of $\hat{u}_t$ on $\hat{u}_{t-1}$.

The most widely used FGLS estimators are the Prais-Winsten (PW) estimator and the Cochrane-Orcutt (CO) estimator. Both apply the two-step OLS estimator (OLS_2) to estimate ρ, then use the estimate $\hat{\rho}$ to quasi-difference the variables Y_t, 1, and t to obtain $\tilde{Y}_t$, $\tilde{1}$, and $\tilde{t}$, where $\tilde{Y}_t = Y_t - \hat{\rho} Y_{t-1}$, $\tilde{1} = 1 - \hat{\rho}$, and $\tilde{t} = t - \hat{\rho}(t-1)$. The CO-FGLS estimators of α and β come from the regression of $\tilde{Y}_t$ on $\tilde{1}$ and $\tilde{t}$ for t = 2, 3, ..., T, ignoring the initial observation. The PW-FGLS estimator for α and β includes the information in the first observation by regressing $\tilde{Y}_t$ on $\tilde{1}$ and $\tilde{t}$ for t = 1, 2, ..., T, where the initial observations are $[Y_1, 1, 1]*(1 - \hat{\rho}^2)^{1/2}$ if $\hat{\rho} < 1$, and $[Y_1, 0, 1]$ if $\hat{\rho} = 1$.

In each of these procedures, the estimator of ρ is obtained by fitting an AR(1) model by OLS, which is the conditional maximum likelihood estimator for Gaussian processes. This estimator differs from the less convenient unconditional maximum likelihood estimator since it ignores the information about ρ contained in the initial observation Y_1, with the difference getting larger as ρ approaches unity. Alternative estimators of ρ have been developed to approximate better the unconditional (quasi-) maximum likelihood estimator of ρ.

Park and Fuller (1994) developed the weighted symmetric least squares (WSLS) estimator of the p-th order autoregressive time series:

$$u_t = \sum_{i=1}^{p} \rho_i u_{t-i} + \varepsilon_t \tag{1.6}$$

where the ε_t's are independent $N(0, \sigma^2)$ random variables.

Note that a stationary time series satisfying (1.6) also satisfies:

$$u_t = \sum_{i=1}^{p} \rho_i u_{t+i} + \eta_t \tag{1.7}$$

where η_t has the same covariance structure as ε_t. The WSLS estimator is obtained by minimizing:

$$Q = \sum_{t=p+1}^{n} w_t [u_t - \sum_{i=1}^{p} \rho_i u_{t-i}]^2 + \sum_{t=1}^{n-p} (1 - w_{t+1})[u_t - \sum_{i=1}^{p} \rho_i u_{t+i}]^2 \tag{1.8}$$

for a given set of weights, $0 \leq w_t \leq 1$ for t = 1, 2, … ,T. In other words, the WSLS estimator is obtained by minimizing the weighted sum of the squares of $\varepsilon_{p+1}, \ldots, \varepsilon_T, \eta_1, \ldots, \eta_{T-p}$. The ordinary least squares estimator is obtained by setting $w_t = 1$. With $w_t = 0.5$, the simple symmetric least squares estimator is obtained. Park and Fuller's (1994) weighted symmetric estimator is obtained by minimizing (1.8) with the following specification of the weights:

$$\begin{aligned} w_t &= 0 & t &= 1, 2, \ldots, p \\ &= (t - p) / (n - 2p + 2) & t &= p+1, \ldots, T-p+1 \\ &= 1 & t &= T-p+2, \ldots, T \end{aligned} \tag{1.9}$$

For the AR(1) error sequence, the WSLS estimator of ρ is:

$$\hat{\rho}_{WS} = [\sum_{t=2}^{T-1} u_t^2 + T^{-1} \sum_{t=1}^{T} u_t^2]^{-1} \sum_{t=2}^{T} u_t u_{t-1} \tag{1.10}$$

The WSLS estimator reduces the downward bias of the OLS estimator of ρ in the AR(1) model, and has a smaller mean squared error for "large" values of ρ. Asymptotically, the two estimators are equivalent.

Fuller (1996) suggested extending the WSLS estimator to the trend model with AR(1) errors by first applying OLS to (1.1) to obtain estimates $\hat{\alpha}_{OLS}$, $\hat{\beta}_{OLS}$, and $\hat{u}_t = Y_t - \hat{\alpha}_{OLS} - \hat{\beta}_{OLS} t$. Then the WSLS estimator $\hat{\rho}_{WS}$ can be computed from (1.10) with $\hat{u}_t$ in place of u_t. Falk

(1999) showed that this estimator of ρ outperforms OLS_1 and OLS_2 in finite samples, especially when ρ is close to or equal to 1. The PW-WSLS estimators for α and β, after quasi-differencing the data, are obtained by regressing $\tilde{Y}_t$ on $\tilde{1}$ and $\tilde{t}$ for t = 1, 2, …, T.

Fuller's (1996) modified weighted symmetric least squares (MWSLS) estimator for the AR(p) model provides an approximately median-unbiased estimator with a root close to or equal to unity.[1] Roy and Fuller (1999) extended the MWSLS estimator to provide an approximately median unbiased estimator of ρ in the linear trend model with AR errors. Their estimator is specified as follows:

$$\hat{\rho}_{MWS} = min(\rho^*, 1) \tag{1.11}$$

where

$$\rho^* = \hat{\rho}_{WS} + C(\hat{\tau}_{WS,1})\hat{\sigma}_{WS}$$
$$\hat{\tau}_{WS,1} = (\hat{\rho}_{WS} - 1)/\hat{\sigma}_{WS}$$

and

$$\begin{aligned} C(\hat{\tau}_{WS,1}) &= -[\tau_{Med} + c_1(\hat{\tau}_{WS,1} - \tau_{Med})] && if\ \hat{\tau}_{WS,1} > \tau_{Med} \\ &= (\hat{\tau}_{WS,1}/T) - 3/[\hat{\tau}_{WS,1} + k(5 + \hat{\tau}_{WS,1})] && if -5 < \hat{\tau}_{WS,1} \le \tau_{Med} \\ &= (\hat{\tau}_{WS,1}/T) - (3/\hat{\tau}_{WS,1}) && if -(3T)^{1/2} < \hat{\tau}_{WS,1} \le -5 \\ &= 0 && if\ \hat{\tau}_{WS,1} \le -(3T)^{1/2} \end{aligned}$$

and τ_{Med} is the median of the limiting distribution of $\hat{\tau}_{WS,1}$ when ρ = 1 (Fuller 1996, Table 10.A.4),

$$k = [3T - \tau^2_{Med}(T + 1)] / [\tau_{Med}(5 + \tau_{Med})(T + 1)]$$
$$c_1 = (1.12 - 1.5/T)/1.65$$

So, to implement the MWSLS estimator of ρ in the AR(1) case, the OLS residuals $\hat{u}_t$ ($= Y_t - \hat{\alpha} - \hat{\beta} t$) are regressed on $\hat{u}_{t-1}$ using the WSLS estimator, which then is adjusted according to (1.11). Then $\hat{\rho}_{MWS}$ is used to quasi-difference the data to get $\widetilde{Y}_t$, $\widetilde{1}$, and $\widetilde{t}$, and the PW-MWSLS estimators for α and β are obtained by regressing $\widetilde{Y}_t$ on $\widetilde{1}$ and $\widetilde{t}$ for t = 1, 2, …, T.

Forecasts generated from the PW-OLS_2, PW-WSLS, and PW-MWSLS estimators (i.e., the Prais-Winsten feasible GLS estimators that use the OLS_2, WSLS, and MWSLS estimators of ρ, respectively), along with those generated from the simple OLS_1 estimator, will be compared in this paper. Forecasts generated from the OLS_2 estimator and CO-FGLS estimators will not be considered because the OLS_1 estimator is better than the OLS_2 estimator and the PW-FGLS estimator is better than the CO-FGLS in the sense of root mean squared errors (RMSEs) in forecasting. Ng and Vogelsang (1999) showed that when ρ is large, forecasts constructed from the OLS_1 estimator have smaller RMSEs than do forecasts constructed from the OLS_2 estimator. They also showed that forecasts constructed from the PW-FGLS estimator have smaller RMSEs than do forecasts constructed from the CO-FGLS estimator.[2]

2.2. Asymptotic Prediction Variance (APV)

This section of the paper considers a couple of methods to compute the asymptotic prediction variance that will be used in constructing prediction intervals. In the appendix we show that for sufficiently large T, following Baillie (1979), the APV of the h-step-ahead prediction error for a linear trend model with AR(1) errors can be expressed as:

$$\begin{aligned} APV_1 = {} & \sigma^2 \sum_{j=0}^{h-1} \rho^{2j} + var(\hat{\alpha})(1 - \rho^h)^2 + var(\hat{\beta})(T + h - \rho^h T)^2 \\ & + 2(1 - \rho^h)(T + h - \rho^h T)\, cov(\hat{\alpha}, \hat{\beta}) \\ & + \sigma^2 \rho^{2(h-1)} h^2 var(\hat{\rho}) / (1 - \rho^2) \end{aligned} \tag{1.12}$$

for $\rho < 1$. When $\rho = 1$, we replace the term $\sigma^2/(1-\rho^2)$ in the last summand by $T\sigma^2$. The first term in (1.12) is due to the random disturbances in the forecast period, the last term is due to the estimation of ρ, and the three terms in the middle arise from the estimation of α and β. Note that the estimator of the h-step-ahead prediction error variance is uncorrelated with the estimators $\hat{\alpha}$ and $\hat{\beta}$, so that covariances involving $\hat{\rho}$ do not appear in (1.12). This APV measure relies on the unknown parameters σ^2 and ρ directly and indirectly (through the variance and covariance terms involving $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\rho}$).

Spitzer and Baillie (1983) proposed a feasible alternative to (1.12) constructed by replacing ρ with $\hat{\rho}$ and σ^2 with $\hat{\sigma}^2$:[3]

$$\begin{aligned} APV_2 = {} & \hat{\sigma}^2 \sum_{j=0}^{h-1} \hat{\rho}^{2j} + var(\tilde{\alpha})(1 - \hat{\rho}^h)^2 + var(\tilde{\beta})(T + h - \hat{\rho}^h T)^2 \\ & + 2(1 - \hat{\rho}^h)(T + h - \hat{\rho}^h T)\, cov(\tilde{\alpha}, \tilde{\beta}) \\ & + \hat{\sigma}^2 \hat{\rho}^{2(h-1)} h^2 var(\tilde{\rho}) / (1 - \hat{\rho}^2) \end{aligned} \tag{1.13}$$

In (1.13), var($\tilde{\alpha}$), var($\tilde{\beta}$), cov($\tilde{\alpha}$,$\tilde{\beta}$), and var($\tilde{\rho}$) are the estimated variances and covariances for $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\rho}$, respectively. When $\hat{\rho} = 1$, we replace the term $\hat{\sigma}^2/(1-\hat{\rho}^2)$ in the last summand by $T\hat{\sigma}^2$.

The computations of APV_1 and APV_2 require the variance and covariance matrix for the coefficient estimators. Since we are using a consistent estimator of ρ, an asymptotically valid estimator of the variance-covariance matrix for $\hat{\alpha}$ and $\hat{\beta}$ can be obtained by treating ρ as known and equal to $\hat{\rho}$ and then estimating the variance-covariance matrix from the following regression:

$$\begin{aligned} Y_t - \hat{\rho}Y_{t-1} &= \alpha(1-\hat{\rho}) + \beta(t - \hat{\rho}(t-1)) + \varepsilon_t \\ &= X_t^*(\hat{\rho})\delta + \varepsilon_t \end{aligned} \tag{1.14}$$

where $X_t^*(\hat{\rho}) = [(1-\hat{\rho})\ (t-\hat{\rho}\ (t-1))]$, which is the row vector of the model matrix $X^*(\hat{\rho})$, and $\delta = [\alpha\ \ \beta]^T$. The resulting estimated covariance matrix is:

$$\frac{SSR(\hat{\delta},\hat{\rho})}{T-k}[\,X^{*T}(\hat{\rho})X^*(\hat{\rho})\,]^{-1} = \begin{bmatrix} \mathrm{var}(\hat{\alpha}) & \mathrm{cov}(\hat{\alpha},\hat{\beta}) \\ \mathrm{cov}(\hat{\beta},\hat{\alpha}) & \mathrm{var}(\hat{\beta}) \end{bmatrix} \tag{1.15}$$

When we compute (1.13) using (1.15) to calculate var($\tilde{\alpha}$), var($\tilde{\beta}$), and cov($\tilde{\alpha},\tilde{\beta}$), setting var($\tilde{\rho}$) = 0, we call the resulting measure of the asymptotic prediction variance $APV_2(1)$.

An alternative procedure that takes into consideration the sampling variance of $\hat{\rho}$ is to use the Gauss-Newton estimator, as suggested by Davidson and MacKinnon (1993):

$$Y_t - \hat{\rho}\,Y_{t-1} - (X_t - \hat{\rho}X_{t-1})\hat{\delta} = (X_t - \hat{\rho}X_{t-1})b + r\hat{u}_{t-1} + \varepsilon_t \tag{1.16}$$

where $\hat{u}_{t-1} = Y_{t-1} - X_{t-1}\hat{\delta}$. Then the estimated covariance matrix for $\hat{\delta}$, i.e., the variances and covariance of $\hat{\alpha}$ and $\hat{\beta}$, will be the upper left-hand 2x2 block of the matrix:

$$\frac{SSR(\hat{\delta},\hat{\rho})}{T-k-1}\begin{bmatrix} X^{*T}(\hat{\rho})X^*(\hat{\rho}) & X^{*T}(\hat{\rho})\hat{u}_{-1} \\ \hat{u}_{-1}^T X^{*T}(\hat{\rho}) & \hat{u}_{-1}^T\hat{u}_{-1} \end{bmatrix}^{-1} \tag{1.17}$$

where $\hat{u}_{-1}$ is the vector containing elements of $\hat{u}_{t-1}$. The right-bottom element is an estimate of the variance of $\hat{\rho}$. Note that the off-diagonal elements of this matrix sequence converge to zero as T increases. Label the computed APV_2 using the variance and covariance estimates of the PW-MWSLS estimators from (1.17) as $APV_2(2)$.

Roy, Falk, and Fuller (1999) also suggested use of the Gauss-Newton estimator. They argued that the ordinary t-statistics for β have large variances for two reasons: first, the variance of the trend coefficient is a highly nonlinear function of ρ; and, second, the maximum variance occurs at $\rho=1$, the boundary of the parameter space, which automatically leads to a negative bias in variance estimation, which in turn is magnified heavily through the estimated variance function that has a very steep slope when ρ is close or near to one. So to use Gauss-Newton estimators will create a test statistic with a distribution that is closer to that of Student's t.

Replacing the variance estimate for $\hat{\rho}$ by the asymptotic variance formula for $\hat{\rho}$:

$$(1 - \hat{\rho}^2)/T \tag{1.18}$$

we calculate APV_2 again with the sampling variance of ρ as shown in (1.18), and label it as $APV_2(3)$.

Then, asymptotically valid $100(1-\alpha)$ percent prediction intervals for the h-step-ahead forecast can be constructed according to

$$\hat{Y}^F_{T+h|T} \pm t_{T-k}(\alpha/2)\sqrt{APV_2} \tag{1.19}$$

where $t_{T-k}(\alpha/2)$ is the $100(1-\alpha/2)$ percentile of the t-distribution with T-k degrees of freedom, and k is the number of parameters used in the estimation.

3. The Monte Carlo Design and Results

Following Falk and Roy (2000), we use the fixed sample size T = 100. For each value of $\rho \in \{0, 0.4, 0.8, 0.9, 0.95, 0.975, 0.99, 1\}$, 1,000 realizations of $\varepsilon_1, \ldots, \varepsilon_{100}, \varepsilon_{101}, \ldots, \varepsilon_{110}$ are generated by independent draws from a standard normal distribution. For each series of ε's, calculate $u_1, \ldots, u_{110}$ and $Y_1, \ldots, Y_{110}$ from equations (1.1) and (1.2) with the restriction that α and β are set equal to zero. The initial value u_0 is drawn from the stationary distribution $N(0, 1/(1-\rho^2))$ for $\rho < 1$, and u_0 is set to 0 when $\rho = 1$.

First, we attempt to replicate the results reported by Falk and Roy (2000) regarding the forecast accuracy for different estimators of the linear trend model. For each sample $Y_1, \ldots, Y_{110}$, the first 100 observations are used to estimate the model according to each of the following previously defined estimators: OLS_1, WSLS, MWSLS, PW-OLS_2, PW-WSLS, and PW-MWSLS. From equations (1.4) and (1.5), $Y^F_{T+h|T}$ and $\hat{Y}^F_{T+h|T}$ are calculated for each estimated model and h = 1, 2, …, 10. The forecast errors that Falk and Roy (1980) are concerned with are attributable to the estimation errors, defined to be the differences between $Y^F_{T+h|T}$ and $\hat{Y}^F_{T+h|T}$. The values of RMSE as a function of ρ and h are reported in Table 1.1.

Results similar to those reported by Falk and Roy are obtained. When ρ is small all the procedures perform about the same; when ρ is sufficiently large, i.e., close to or equal to one, the PW-MWSLS procedure forecasts better in the RMSE sense than the other procedures. However, when ρ is in the neighborhood of 0.8 to 0.9, the PW-MWSLS estimator actually produces higher RMSEs than the PW-FGLS and PW-WSLS procedures, particularly at longer forecast horizons. The explanation given by Falk and Roy (1999) is that the FGLS estimators of the trend coefficient β that use the median-unbiased estimators of ρ (PW-

MWSLS) will have higher variances than the downward-biased estimators of ρ (PW-FGLS or PW-WSLS) when ρ is sufficiently large but not too close to one. Overall, in terms of RMSE, the PW-MWSLS procedure is recommended by Falk and Roy as the best procedure to apply in forecasting from the trend model with autoregressive errors, and their conclusion is confirmed here.

This paper extends Falk and Roy's paper by comparing the nominal and actual coverage properties of prediction intervals constructed from the various Prais-Winsten estimators of the linear trend model with autoregressive errors. Simulation methods are used to see which estimator's prediction intervals give the probability of coverage that is closest to the asymptotic (i.e., nominal) probability of coverage. Following the same simulation design that was applied to construct the results shown in Table 1.1, 90 percent prediction intervals for the h-step-ahead predictions are constructed using equation (1.19) and the asymptotic prediction variance measures $APV_2(1)$, $APV_2(2)$, and $APV_2(3)$, respectively. Tables 1.2, 1.3, and 1.4 provide the actual percentages of times that $Y_{101}, \ldots, Y_{110}$ fell into these intervals.

Table 1.2 shows the probability of coverage when $APV_2(1)$, which ignores the sampling variability of ρ, is used to compute the asymptotic prediction measure from the PW-OLS_2, PW-WSLS, and PW-MWSLS estimators. The coverage rates for these three estimators are clustered tightly around 90 percent for h = 1, 2, and ρ = 0.0, 0.4, ranging between 88.6 percent and 91.7 percent. As ρ and/or h increases, the coverage rates corresponding to the PW-OLS_2 and PW-WSLS estimators decrease substantially. However, the coverage rates corresponding to PW-MWSLS remain quite stable, falling within 88.1 percent and 92.1 percent, except for the following three cases: ρ = 0.975 and h = 10 (0.869), ρ = 1 and h = 8

(0.862), and $\rho = 1$ and h = 10 (0.836). Even at the values of these exceptions, however, we can see that they still perform much better than PW-OLS$_2$ and PW-WSLS, by being from 6.8 percent to 8.9 percent closer to the true coverage rate of 90 percent. The exceptions all occur at long horizons, where it is reasonable to expect deteriorating results.

Table 1.3 uses the asymptotic prediction variance APV$_2$(2), which explicitly accounts for the sampling variability in estimating ρ, to construct 90 percent prediction intervals from the PW-OLS$_2$, PW-WSLS, and PW-MWSLS estimators. The coverage rates for the PW-OLS$_2$ and PW-WSLS estimators improve slightly in comparison to the coverage rates reported in Table 1.2, but the qualitative conclusions drawn from Table 1.2 also apply to Table 1.3. Specifically, the actual coverage rates are well below 90 percent for large ρ and/or large h. The coverage rates for the PW-MWSLS estimator improve slightly when ρ = 0.0, 0.4, 0.8, and 0.9, although in both tables these coverage rates are very close to 90 percent. There is also some improvement when $\rho = 1$. Note in particular that when $\rho = 1$ and h = 8, and when $\rho = 1$ and h = 10, using APV$_2$(2) rather than APV$_2$(1) improves the coverage of the prediction intervals for PW-MWSLS considerably. However, for ρ = 0.95, 0.975, and 0.99, and when h = 1, 2, and 4, the coverage rates from the PW-MWSLS estimator generally are further away from 90 percent when APV$_2$(2) is applied than when APV$_2$(1) is applied. The reason for this is unclear and may warrant further study.

We also computed the 90 percent prediction interval from PW-OLS$_2$, PW-WSLS, and PW-MWSLS estimators using the asymptotic prediction variance APV$_2$(3) when the variance estimate for $\hat{\rho}$ is derived by the asymptotic variance formula for $\hat{\rho}$ in equation (1.18). The conclusions about these three estimators are quite similar to the results shown earlier in Table

1.3, so those results are not reported in this paper. Such similar results provide an alternative way to construct the asymptotic prediction interval, which will give coverage properties as good as those obtained using $APV_2(2)$.

4. Bootstrap Methods and Results

As pointed out by Clements and Taylor (2001), there are a number of issues on the calculation of prediction intervals with appropriate coverage levels, which include the need to allow for parameter estimation uncertainty, small-sample biases of the parameter estimates, need to take non-linearity into consideration when the forecasts are non-linear functions of the parameter estimates, etc. The bootstrap method may work better than simulations by re-sampling to avoid inaccurate approximations to biases, variances, and other measures of uncertainty. In this section, the bootstrap method is applied to verify the results by simulation, and in the hope of obtaining results that would improve the simulation results presented in the previous section.

The bootstrap procedure is as follows:

Step (1):

For fixed sample size T = 100, and for each value of $\rho \in \{0, 0.4, 0.8, 0.9, 0.95, 0.975, 0.99, 1\}$, we generate series $\{\varepsilon_t\}_{t=1}^{110}$ by independent draws from a standard normal distribution, and then calculate series $\{u_t\}_{t=1}^{110}$ and $\{Y_t\}_{t=1}^{110}$ from equations (1.1) and (1.2) with the restriction that α and β are set equal to zero. The initial value u_0 is drawn from the stationary distribution $N(0, 1/(1-\rho^2))$ for $\rho < 1$, and u_0 is set to 0 when $\rho = 1$.

Step (2):

For the sample $\{Y_t\}_{t=1}^{110}$, the last 10 observations are set aside for the purpose of forecasting, and the first 100 observations are used to estimate the model. The final estimates $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\rho}$ are obtained using each of the three different estimators: PW-OLS$_2$, PW-WSLS, and PW-MWSLS. The residual series is calculated as $\hat{u}_t = \widetilde{Y}_t - \hat{\alpha}\,\widetilde{1} - \hat{\beta}\,\widetilde{t}$ for t = 1, ... , 100. The following steps are introduced by focusing on one estimator to avoid confusion.

Step (3):

Generate B=1000 bootstrap samples u_1^b, ... , u_{100}^b by sampling with replacement from $\{\hat{u}_t\}_{t=1}^{100}$. For each bootstrap sample, compute $\hat{a} = \hat{\alpha}(1-\hat{\rho})+\hat{\beta}\hat{\rho}$ and $\hat{b} = \hat{\beta}(1-\hat{\rho})$, then construct $\{Y_t^b\}$ series using the formula $Y_t^b = \hat{a} + \hat{b}\,\mathrm{t} + \hat{\rho}\,Y_{t-1}^b$ for t = 2, ... ,100, and the initial value Y_1^b is set to be u_1^b; i.e., we treat the initial value as fixed when we construct the bootstrap samples.

Step (4):

For each bootstrap sample Y_1^b, ... , Y_{100}^b, the observations are used to estimate the model. The estimate $\hat{\rho}^b$ is then applied to quasi-difference the variables Y_t^b, 1 and t to obtain $\widetilde{Y}_t^b$, $\widetilde{1}^b$, and $\widetilde{t}^b$, where $\widetilde{Y}_t^b = Y_t^b - \hat{\rho}^b\,Y_{t-1}^b$, $\widetilde{1}^b = 1 - \hat{\rho}^b$, $\widetilde{t}^b = \mathrm{t} - \hat{\rho}^b(\mathrm{t}-1)$ for t = 1, ... , T.

Step (5):

The h-step-ahead forecast is computed using the formula

$$\hat{Y}_{T+h}^b = \hat{\alpha}^b + \hat{\beta}^b(\mathrm{T}+\mathrm{h}) + (\hat{\rho}^b)^h\,\hat{u}_T^b, \qquad \mathrm{h} = 1, \ldots, 10 \tag{1.20}$$

where $\hat{u}_T^b = \hat{Y}_t^b - \hat{\alpha}^b - \hat{\beta}^b\,\mathrm{T}$.

Step (6):

The bootstrap-t method is applied to obtain the 100*(1-α) percent prediction interval for the h-step-ahead forecast, which can be constructed according to

$$(\hat{Y}^F_{T+h|T} - \hat{t}_{(1-\alpha)}\sqrt{APV_2}\ ,\ \hat{Y}^F_{T+h|T} - \hat{t}_{(\alpha)}\sqrt{APV_2}\ ) \tag{1.21}$$

In this paper, we computed 90 percent prediction interval, so (1-α) = 0.95, and α = 0.05; then the estimate of the 5 percent point is the 50th largest value of the Z^*(b)s and the estimate of the 95 percent point is the 950th largest value of the Z^*(b)s, with

$$Z^*(b) = (\hat{Y}^b_{T+h|T} - \hat{Y}^F_{T+h|T})\ /\ \hat{se}^*(b) \tag{1.22}$$

where the estimate of the standard error for $\hat{Y}^b_{T+h|T}$ is

$$\hat{se}^*(b) = \{\textstyle\sum_{b=1}^{B}[\hat{Y}^b_{T+h|T} - \hat{Y}^{*b}_{T+h|T}]^2/(\mathrm{B}-1)\}^{1/2} \tag{1.23}$$

and

$$\hat{Y}^{*b}_{T+h|T} = \textstyle\sum_{b=1}^{B}\hat{Y}^b_{T+h|T}\ /\ B \tag{1.24}$$

Step (7):

Repeat the above steps (1) to (6) 1000 times, then count the number of times that the true value for the h-step ahead Y_{T+h} value falls in the prediction interval that uses formula (1.21). The results for the actual percentages of times that Y_{101}, … , Y_{110} fell into these prediction intervals for each of the Prais-Winsten estimators are reported in Tables 1.4 and 1.5.

Table 1.4 shows the probability of coverage when the asymptotic prediction variance measure $APV_2(1)$, ignoring the sampling variance in ρ, is used in the calculation of the prediction interval. Results similar to those of simulation are obtained even though the numbers are smaller in general. For ρ = 0.0 and 0.4, the coverage rates for the three estimators PW-OLS_2, PW-WSLS, and PW-MWSLS perform about the same; the largest difference is about 0.02. As ρ and/or h increases, the coverage rates corresponding to PW-OLS_2 and PW-WSLS decrease substantially but PW-WSLS performs better than PW-OLS_2. For $\rho = 0.99$ and 1.0 and $h = 10$, the coverage rate is 0.693 and 0.714, respectively, when the PW-OLS_2 estimator is used; and the coverage rate is 0.745 and 0.747, respectively, when the PW-WSLS estimator is used. The coverage rates when the PW-MWSLS estimator is applied remain quite stable: for $\rho = 0.975$, 0.99, and 1.0 across all steps, the coverage rates falls within 86.0 percent and 90.2 percent.

The coverage rates using the asymptotic prediction variance $APV_2(2)$ in the calculation of the prediction interval are presented in Table 1.5. The same qualitative results are obtained as those of using the simulation method. The coverage rates for PW-OLS_2 and PW-WSLS improve slightly, compared to the coverage rates calculated when ignoring the variance in ρ; and the coverage rates are still well below the 90 percent when ρ is close to 1 and for large h; for example: $\rho = 0.99$ and $h = 8$ (0.732) if the PW-OLS_2 estimator is used, and in contrast $\rho = 1.0$ and $h = 10$ (0.762) if use the PW-WSLS estimator. The coverage rates for the PW-MWSLS estimator improve when $\rho = 0.0$, 0.4, 0.8, and 0.95; the values are closer to the true 90 percent if we use $APV_2(2)$ as the asymptotic variance rather than $APV_2(1)$. When $\rho = 0.975$, 0.99, and 1.0, the coverage rates are above 90 percent compared to the results in Table 1.4; especially for $h = 4$, 8, and 10, the coverage rates generally are farther away from 90 percent when $APV_2(2)$ is applied than when $APV_2(1)$ is applied; but these results are still much closer to 90 percent than those coverage rates when PW-OLS_2 and/or PW-WSLS estimators are used. Note that when $h = 1$ and $\rho = 0.0$, 0.4, and 0.8, the coverage rates are not as close to 90 percent as are those results in the simulation; these results may be improved if we use different starting values in constructing bootstrap series, or if we use a different bootstrap method, like the bias-corrected and accelerated (BC_a) bootstrap.

In summary, the results using the bootstrap confirms that the use of the PW-MWSLS estimator improves the coverage, compared to using PW-OLS_2 and PW-WSLS estimators in general when the time series data are best represented by a linear trend model with autoregressive errors.

5. Empirical Example

We turn to the empirical examples in this section. Many time series can be modeled as a linear trend model with autoregressive errors. Ng and Vogelsang (1999) treated seven U.S. macroeconomic time series (GDP, investment, exports, imports, final sales, personal income, and employee compensation) as linear trend models with AR(4) errors. Roy, Falk, and Fuller (1999) modeled U.S. gross national product data as a linear trend model with AR(2) errors, and they also modeled interest rate series assumed to follow a linear trend model with AR(3) errors in their examples. In our paper, we use the variable real gross domestic product (RGDP) as the empirical example because it tends to grow over time, which makes it more suitable for a linear trend model. The countries we picked are: Canada, France, Germany, Italy, the United Kingdom and the United States.

First, we analyze the data for each country and decide whether the model of linear trend with autoregressive errors is suitable for the series. After choosing and fitting the optimal model, we compute the forecasting values and construct the prediction interval using the PW-

MWSLS estimator. The data are all quarterly data, obtained from *International Financial Statistics*. The ranges of the data are as follows: Canada and the United Kingdom (1957:1-1999:3), France (1970:1-1999:2), Germany (1960:1-1998:4), Italy (1960:1-1998:4), and the United States (1957:1-1999:1). All RGDP series are transformed to natural logs, and labeled as LGDP. The software WINRATS is used to conduct the analysis.

First, we plot the transformed RGDP series in Figure 1.1. A clear upward trend can be observed for all countries. So, it is reasonable to assume that LGDP for each country has a linear trend. Further, we assume that the error term is serially autocorrelated, which usually is true for economic time series data. Therefore, the LGDP is assumed to be a linear trend model with AR(p) errors, and has the form:

$$\begin{aligned} LGDP_t &= \alpha + \beta t + u_t \qquad t = 1,2,\ldots,T \\ u_t &= \sum_{i=1}^{p} \rho u_{t-i} + \varepsilon_t \end{aligned} \tag{1.25}$$

So we start by regressing LGDP on a constant and a trend term. For the purpose of forecasting, we set aside ten observations for each LGDP series; for example, the data from 1996:4 to 1999:1 are set aside for the U.S. LGDP series, the total number of observations we used in the analysis is T = 159 observations, rather than T = 169. The ordinary least squares estimates of α and β are obtained, and the error series $\{\hat{u}_t\}$ is obtained by

$$\hat{u}_t = LGDP_t - \hat{\alpha}_{OLS} - \hat{\beta}_{OLS}\, t \qquad t = 1, \ldots, T \tag{1.26}$$

Next, we check the autocorrelations and the partial autocorrelations of the error series $\{\hat{u}_t\}$. The estimated coefficients of the first twelve values of ACF and PACF are reported in Table 1.6. As we can see, the autocorrelation for all countries starts at relatively high values at lag one, and decays slowly with increasing lags; the sample ACFs' shown in Figure 1.2

demonstrate the same slow decaying. The sample PACFs in Figure 1.2 can be used to find the approximate AR orders of the error series $\{\hat{u}_t\}$. To be more specific and accurate, the Box-Jenkins procedure is applied, the AIC (Akaike Information Criterion) and SBC (Schwartz Bayesian Criterion) are used to select the best AR model. The chosen AR model for the series $\{\hat{u}_t\}$ for each country and the AIC and SBC values are presented in Table 1.7. Both the AIC and SBC picked the same model for France, Germany and Italy; for Canada, the United Kingdom and the United States, the AIC and SBC choose a different model. We will pursue our study using the model chosen by SBC first, since SBC usually picks out the most parsimonious model. So we fit AR(1) for the error series $\{u_t\}$ for Italy and the United Kingdom, AR(2) for Canada and the United States, AR(3) for France, and AR(5) for Germany.

Fitting the chosen AR model to each of the error series $\{\hat{u}_t\}$ of the linear trend model, the residual $\{\hat{\varepsilon}_t\}$ is obtained from the fitted model:

$$\hat{\varepsilon}_t = \hat{u}_t - \sum_{i=1}^{p} \hat{\rho}_i \hat{u}_{t-i} \qquad t = 1, \dots, T \tag{1.27}$$

Our model setup is that $\{\varepsilon_t\}$ is uncorrelated random variable. So, we need to verify there is no autocorrelation in $\{\hat{\varepsilon}_t\}$. Proceed, as in the case for $\{\hat{u}_t\}$, to check the autocorrelations and the partial autocorrelations of the series $\{\hat{\varepsilon}_t\}$. The results show that for Canada, Italy, and the United Kingdom, there exist serial autocorrelation in the residual $\{\hat{\varepsilon}_t\}$. So for these countries' error series $\{\hat{u}_t\}$, we fit the best model chosen by AIC again, and the residual series $\{\hat{\varepsilon}_t\}$ show no autocorrelation this time. Therefore, the order of AR model for $\{\hat{u}_t\}$ for all three countries is p = 4.

Fit the linear trend model with autoregressive error term for each of the six countries, the estimated coefficients of the first twelve values of the ACF and PACF for the residual series $\{\hat{\varepsilon}_t\}$ are reported in Table 1.8. The Ljung-Box Q statistics of the residuals $\{\hat{\varepsilon}_t\}$ indicate that as a group, lags 1 through 6, 6 through 12, up to 18 through 24, all are not significantly different from zero, meaning there do not appear to be any serial correlations in $\{\hat{\varepsilon}_t\}$ for each of the six countries; and the individual tests on the coefficients of ACF show the same results.

We apply the PW-MWSLS estimator on the LGDP data for Canada, France, Germany, Italy, the United Kingdom, and the United States. For the United States, the LGDP series follows a linear trend model with AR(2) errors, i.e., model (1.25), is assumed to be p = 2. We need a different weighted symmetric regression than one with AR(1) errors, so we follow the procedure suggested by Roy, Falk, and Fuller (1999), and we use the U.S. LGDP data to demonstrate how to find the PW-MWSLS estimates for the parameters and obtain forecasts. For the higher order of AR errors, it is a straightforward extension.

Note that the linear trend model with AR(2) can be rewritten as

$$\begin{aligned} LGDP_t &= \alpha + \beta t + u_t && t = 1,2,\ldots \\ u_t &= \rho u_{t-1} + \psi\, \Delta u_{t-1} + \varepsilon_t && \rho \in (-1,1] \end{aligned} \tag{1.28}$$

First, after de-trending the U.S. LGDP series using the OLS estimates of α and β, we get $\hat{u}_t$, as shown in (1.26).

Let $\hat{\rho}_{WS}$ denote the trend-adjusted WSLS estimator of ρ obtained by the weighted symmetric regression of $\hat{u}_t$ on $\hat{u}_{t-1}$ and $\Delta\hat{u}_{t-1}$, $\Delta\hat{u}_{t-2}$, … , $\Delta\hat{u}_{t-p+1}$ where $\Delta\hat{u}_t = \hat{u}_t - \hat{u}_{t-1}$, and p is the order of the AR model. The fitted weighted symmetric autoregressive equation for the U.S. LGDP data is (with standard errors in parentheses):

$$\underset{}{\hat{u}_t} = \underset{(0.01639)}{0.9691\,\hat{u}_{t-1}} + \underset{(0.07594)}{0.3180\,\Delta\hat{u}_{t-1}} \tag{1.29}$$

Then $\hat{\rho}_{WS}$ is adjusted according to the formulas shown in (1.11), and the modified WSLS estimator of ρ after the adjustment is $\hat{\rho}_{MWS}$ = min(ρ^*, 1) = 1. Next, ψ is re-estimated by using an OLS regression of $\hat{u}_t$ - $\hat{\rho}_{MWS}$ $\hat{u}_{t-1}$ on $\Delta\hat{u}_{t-1}$, and $\hat{\psi}$ = 0.3018.

The previous theoretical part gives the PW-FGLS transformation for the linear trend model with AR(1) error. In the case of U.S. LGDP, the linear trend model has AR(2) errors, so we quasi-difference the variables LGDP, 1, and t as follows:

$$\begin{aligned} LG\tilde{D}P_t &= LGDP_t - \hat{\rho}_{MWS} LGDP_{t-1} - \hat{\psi}\,\Delta LGDP_t \\ \tilde{1}_t &= 1 - \hat{\rho}_{MWS} \\ \tilde{t}_t &= t - \hat{\rho}_{MWS}(t-1) - \hat{\psi} \end{aligned} \tag{1.30}$$

for t = 3, 4, …, T; the initial values are set as $LG\tilde{D}P_1 = LG\tilde{D}P_2 = 0$, $\tilde{1}_1$ = $\tilde{1}_2$ = 0, and $\tilde{t}_1 = \tilde{t}_2 = 1 - \hat{\psi}$. The PW-FGLS estimate for α is $\hat{\alpha}_{MWS}$ = 0 and for β is $\hat{\beta}_{MWS}$ = 0.0077.

The estimated forecast value for the LGDP using the PW-MWSLS estimator is constructed as:

$$LG\tilde{D}P_{T+h|T} = \hat{\alpha}_{MWS} + \hat{\beta}_{MWS}(T+h) + \hat{u}_{T+h} \tag{1.31}$$

where

$$\begin{aligned} \hat{u}_{T+h} &= \theta_1\,\hat{u}_T + \theta_2\,\hat{u}_{T-1} && \text{if } h = 1 \\ &= \theta_1\,\hat{u}_{T+1} + \theta_2\,\hat{u}_T && \text{if } h = 2 \\ &= \theta_1\,\hat{u}_{T+h-1} + \theta_2\,\hat{u}_{T+h-2} && \text{if } h = 3,\ldots,10 \end{aligned}$$

and

$$\theta_1 = \hat{\rho}_{MWS} + \hat{\psi}$$
$$\theta_2 = -\hat{\psi}$$

To construct the prediction interval for the h = 1, 2, ..., 10 step-ahead forecast for the LGDP series for each country after we get the estimates for parameters in the model and the forecast values, we calculate the values for $APV_2(1)$ and $APV_2(2)$; note that since the formula for APV is given for the case of AR(1) error, for the U.S., we treat $\hat{\psi}$ as fixed in this paper, which allows us to use the asymptotic prediction variance formula directly. The true value of the LGDP, together with the lower bound and the higher bound of the prediction interval for forecasted estimates of LGDP for each of the six countries, are given in Table 1.9. The length of each interval also is provided in the same table. The forecast values for all step-ahead values of LGDP are quite close to the actual value of LGDP we set aside in the beginning, and all lie within the asymptotic prediction intervals. The length of the prediction interval using $APV_2(2)$ is greater than the length of the prediction interval using $APV_2(1)$. This is reasonable because the former takes into account the sampling variance in the estimate of $\hat{\rho}$, and the larger the length of the prediction interval the higher the coverage rates. As we demonstrated in the simulation test, the prediction interval using $APV_2(2)$ gives values much closer to the true probability coverage as ρ approaches 1, compared to the probability of coverage using other estimators, including $APV_2(1)$, especially when the forecast horizon increases.

Besides the above examples of predicting the future one- to ten-step-ahead forecasts and constructing the asymptotic prediction intervals, following Ng and Vogelsang (1999), we calculate the RMSE for 100 one-period-ahead forecasts using PW-OLS_2 and PW-MWSLS estimates, respectively. For each estimator, we set aside 100 observations first (except for

France and Italy, we set aside 50 observations due to fewer available data). For example, the available U.S. RGDP data span 1957:1-1999:1, the first forecast we get is based on estimation up to 1974:1, and the RMSE is calculated; then we add the next observation from the available data set, and the second forecast is obtained together with the RMSE based on the extended data set. We continue updating the data set until 1999:1, and get the last one-step-ahead forecast and the RMSE. After we get 100 RMSEs, we calculate the mean of these RMSEs. The results are consistent with our simulation results, when the error series from the linear trend model has a ρ value close to or equal to unity, the PW-MWSLS estimator is better than the PW-OLS_2 estimator in having a smaller RMSE:

	PW-OLS_2	PW-MWSLS		PW-OLS_2	PW-MWSLS
Canada	0.0018	0.0004	Italy	0.0334	0.0020
France	0.0145	0.0011	U.K.	0.0022	0.0009
Germany	0.0051	0.0020	U.S.	0.0088	0.0001

Again, we showed that the forecasts based on PW-MWSLS yield more accurate prediction values.

6. Conclusion

This paper has extended work by Falk and Roy (2000). They focused on a Monte Carlo comparison of the out-of-sample forecasting performances of the OLS, WSLS, MWSLS, PW-OLS, PW-WSLS, and PW-MWSLS estimators of the linear trend model with autoregressive and possibly unit-root errors. Using the RMSE standard, Falk and Roy concluded that the PW-MWSLS estimation method is the best procedure to use to construct point forecasts. We use Monte Carlo methods to compare the actual coverage rates of nominal 90 percent prediction intervals constructed from the PW-OLS_2, PW-WSLS, and PW-MWSLS estimators in an effort to see whether the best point predictor also leads to prediction intervals with the most accurate coverage rates, and the bootstrap method is applied to verify the results in a simulation.

Our main conclusions are as follows. First, as the autoregressive coefficient ρ increases or the forecast horizon h increases, the actual coverage rates of the forecast intervals constructed from the PW-OLS_2 and PW-WSLS estimators fall substantially below the nominal rate. This conclusion does not depend on whether sampling variability in the estimate of ρ is accounted for in computing the asymptotic prediction variance. Second, when the sampling variability of ρ is ignored in computing the asymptotic prediction variance the forecast intervals constructed from the PW-MWSLS estimator generally have very good coverage rates. The main exceptions occur when ρ is equal to one and the forecast horizon is large, although even in these cases the coverage rates are mostly greater than 85 percent. Finally, when ρ is equal to one, the coverage rates for forecast intervals constructed from the PW-MWSLS estimator improve substantially if sampling variability in the estimate of ρ is accounted for in computing the asymptotic prediction variance. However, the coverage rates when ρ is close to but less than one deteriorate but still are much better than those of other estimators.

Overall we conclude that point forecasts and forecast intervals constructed from the PW-MWSLS estimator are preferred to those constructed from OLS or other Prais-Winsten estimators. These forecast intervals appear to have good coverage properties whether or not sampling variability in the estimate of ρ is accounted for in computing the asymptotic prediction variance. The bootstrap method does not improve the results by simulation as we had thought, but future studies may improve our results by applying different bootstrap methods.

Empirical examples are given about how to use the PW-MWSLS estimator to forecast and construct the prediction interval. The estimated forecast values for the logarithm of national gross domestic product for six countries (Canada, France, Germany, Italy, the United Kingdom, and the United States) are quite close to the actual values, and the lengths of the prediction intervals are reasonable. As we know from the simulation results, these estimates provide an approximately median-unbiased estimator since the root is equal to unity, and the prediction intervals give good coverage properties. Thus, we can rely on the PW-MWSLS method to uncover the dynamic path of the economic time series, and based on the forecasted future values can conduct economic and production planning or policy evaluations.

Notes

1. Andrews (1993) developed an exact median-unbiased estimator for the first-order AR model, which was extended by Andrews and Chen (1994) to an approximately median-unbiased estimator for the p-th order AR model.

2. Canjels and Watson (1997) argued that the PW-FGLS estimators are preferred to the CO-FGLS estimators in the linear trend model with AR(1) errors that can have a unit root.

3. Spitzer and Baillie (1983), relying on the consistency of $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\rho}$, also considered assuming that the prediction error variance is entirely due to the random shocks that occur during the forecast period, so that $APV_3 = \hat{\sigma}^2 \sum_{j=0}^{h-1} \hat{\rho}^{2j}$. The probabilities of the coverage for the prediction interval when the above formula is used are not reported because the results are similar to those reported in Table 1.2. Results are available upon request.

Appendix

Derivation of the formula for asymptotic prediction variance (APV):

The linear trend model with AR(1) error is:

$$Y_t = \alpha + \beta t + u_t = X_t\delta + u_t \tag{A.1}$$

where $u_t = \rho u_{t-1} + \varepsilon_t$, ε_t is uncorrelated with mean zero and variance σ^2:

$$X_t = [1 \;\; t],\ \delta^T = [\alpha \;\; \beta]$$

Rewriting the $\{u_t\}$ sequence as an infinite moving average representation, we have:

$$u_t = \rho u_{t-1} + \varepsilon_t = \rho(\rho u_{t-2} + \varepsilon_{t-1}) + \varepsilon_t = \rho^2 u_{t-2} + \varepsilon_t + \rho\varepsilon_{t-1} = \ldots$$

$$= \sum_{j=0}^{h-1} \rho^j \varepsilon_{t-j} + \rho^h u_{t-h} \tag{A.2}$$

Now, substitute u_t into (A.1):

$$Y_t = X_t\delta + \sum_{j=0}^{h-1} \rho^j \varepsilon_{t-j} + \rho^h u_{t-h}$$

$$= X_t\delta + \sum_{j=0}^{h-1} \rho^j \varepsilon_{t-j} + \rho^h(Y_{t-h} - X_{t-h}\delta) \tag{A.3}$$

Then, replacing t with T + h in (A.3), we obtain:

$$Y_{T+h} = \sum_{j=0}^{h-1} \rho^j \varepsilon_{T+h-j} + X_{T+h}\delta + \rho^h(Y_T - X_T\delta) \tag{A.4}$$

The parameters generally are unknown in practice, so the estimated h-step-ahead forecast is

$$\hat{Y}_{T+h} = X_{T+h}\hat{\delta} + \hat{\rho}^h(Y_T - X_T\hat{\delta}) \tag{A.5}$$

Then the h-step-ahead prediction error, e_{T+h}, is defined as the difference between (A.4) and (A.5):

$$e_{T+h} = Y_{T+h} - \hat{Y}_{T+h}$$

$$= [\sum_{j=0}^{h-1} \rho^j \varepsilon_{T+h-j} + X_{T+h}\delta + \rho^h(Y_T - X_T\delta)] - [X_{T+h}\hat{\delta} + \hat{\rho}^h(Y_T - X_T\hat{\delta})]$$

$$= \sum_{j=0}^{h-1} \rho^j \varepsilon_{T+h-j} - X_{T+h}(\hat{\delta} - \delta) + (\rho^h - \hat{\rho}^h)\, Y_T - \rho^h X_T\delta + \hat{\rho}^h X_T\hat{\delta} \tag{A.6}$$

Using the first-order Taylor expansion:

$$\hat{\rho}^h Y_T = \rho^h Y_T + h\, \rho^{h-1} Y_T(\hat{\rho} - \rho)$$

$$\hat{\rho}^h X_T \hat{\delta} = \rho^h X_T \delta + \rho^h X_T (\hat{\delta} - \delta) + (h\rho^{h-1}) X_T \delta (\hat{\rho} - \rho)$$

We then can rewrite e_{T+h} as:

$$e_{T+h} = \sum_{j=0}^{h-1} \rho^j \varepsilon_{T+h-j} - X_{T+h}(\hat{\delta} - \delta) - h\rho^{h-1} Y_T(\hat{\rho} - \rho) + \rho^h X_T(\hat{\delta} - \delta) + h\rho^{h-1} X_T \delta(\hat{\rho} - \rho)$$

$$= \sum_{j=0}^{h-1} \rho^j \varepsilon_{T+h-j} + (\rho^h X_T - X_{T+h})(\hat{\delta} - \delta) + h\rho^{h-}1(X_T\delta - Y_T)(\hat{\rho} - \rho)$$

$$= \sum_{j=0}^{h-1} \rho^j \varepsilon_{T+h-j} + (\rho^h - 1)(\hat{\alpha} - \alpha) + (\rho^h T - T - h)(\hat{\beta} - \beta) + h\rho^{h-1}(X_T\delta - Y_T)(\hat{\rho} - \rho)$$

Then the asymptotic prediction variance is given by:

$$APV = \sigma^2 \sum_{j=0}^{h-1} \rho^{2j} + \mathrm{var}(\hat{\alpha})(1-\rho^h)^2 + \mathrm{var}(\hat{\beta})(T + h - \rho^h T)^2$$
$$+ 2(1-\rho^h)(T + h - \rho^h T)\,\mathrm{cov}(\hat{\alpha}, \hat{\beta}) + h^2 \rho^{2(h-1)}\,\mathrm{var}(\hat{\rho})\,\mathrm{var}(u_T)$$

where

$$\mathrm{var}(u_T) = \mathrm{var}(Y_T - X_T\delta)$$
$$= \sigma^2/(1 - \rho^2) \qquad \rho < 1$$
$$= T\sigma^2 \qquad \rho = 1 \qquad \text{(A.7)}$$

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Table 1.1. RMSEs for h-step-ahead forecasts in linear trend model with AR(1) errors

One-Step Ahead RMSEs:

Rho	OLS-1	WSLS	MWSLS	PW-OLS2	PW-WSLS	PW-MWSLS
`0.00	0.238	0.234	0.234	0.236	0.236	0.236
0.40	0.243	0.243	0.237	0.240	0.240	0.238
0.80	0.260	0.270	0.249	0.253	0.254	0.251
0.90	0.272	0.294	0.256	0.263	0.264	0.252
0.95	0.282	0.318	0.240	0.271	0.269	0.228
0.975	0.286	0.328	0.221	0.270	0.264	0.201
0.99	0.273	0.320	0.197	0.259	0.253	0.180
1.00	0.251	0.303	0.168	0.238	0.230	0.139

Two-Step Ahead RMSEs:

Rho	OLS-1	WSLS	MWSLS	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.216	0.214	0.214	0.214	0.214	0.214
0.40	0.314	0.311	0.310	0.309	0.309	0.311
0.80	0.446	0.460	0.435	0.434	0.435	0.442
0.90	0.498	0.534	0.477	0.480	0.482	0.474
0.95	0.532	0.594	0.460	0.507	0.504	0.439
0.975	0.544	0.621	0.428	0.510	0.501	0.390
0.99	0.522	0.610	0.383	0.492	0.479	0.350
1.00	0.479	0.577	0.324	0.449	0.434	0.269

Four-Step Ahead RMSEs:

Rho	OLS-1	WSLS	MWSLS	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.222	0.219	0.219	0.220	0.220	0.219
0.40	0.359	0.356	0.355	0.354	0.354	0.355
0.80	0.698	0.709	0.691	0.675	0.677	0.709
0.90	0.859	0.906	0.846	0.821	0.825	0.848
0.95	0.963	1.059	0.854	0.907	0.904	0.823
0.975	1.003	1.131	0.811	0.930	0.915	0.745
0.99	0.971	1.121	0.731	0.899	0.878	0.672
1.00	0.886	1.057	0.614	0.815	0.790	0.511

Eight-Step Ahead RMSEs:

Rho	OLS-1	WSLS	MWSLS	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.234	0.232	0.232	0.232	0.232	0.232
0.40	0.387	0.383	0.383	0.380	0.380	0.380
0.80	0.971	0.969	0.964	0.935	0.936	0.993
0.90	1.371	1.408	1.383	1.295	1.301	1.403
0.95	1.660	1.772	1.519	1.536	1.534	1.482
0.975	1.785	1.958	1.495	1.621	1.603	1.394
0.99	1.753	1.968	1.371	1.575	1.545	1.272
1.00	1.588	1.842	1.139	1.407	1.369	0.957

Ten-Step Ahead RMSEs:

Rho	OLS-1	WSLS	MWSLS	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.241	0.238	0.238	0.238	0.238	0.238
0.40	0.398	0.393	0.393	0.390	0.390	0.390
0.80	1.053	1.046	1.043	1.012	1.013	1.073
0.90	1.566	1.591	1.585	1.473	1.479	1.615
0.95	1.959	2.061	1.806	1.798	1.798	1.772
0.975	2.138	2.312	1.810	1.924	1.905	1.698
0.99	2.114	2.336	1.674	1.874	1.841	1.561
1.00	1.910	2.179	1.387	1.662	1.620	1.170

Table 1.2. Simulation results of the probability of coverage using $APV_2(1)$

One-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.905	0.889	0.897
0.40	0.895	0.901	0.905
0.80	0.893	0.889	0.899
0.90	0.885	0.896	0.894
0.95	0.902	0.886	0.897
0.975	0.898	0.869	0.919
0.99	0.896	0.870	0.921
1.00	0.890	0.879	0.892

Two-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.895	0.904	0.886
0.40	0.899	0.898	0.911
0.80	0.886	0.885	0.907
0.90	0.855	0.883	0.902
0.95	0.894	0.882	0.903
0.975	0.878	0.860	0.894
0.99	0.872	0.865	0.916
1.00	0.855	0.869	0.884

Four-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.894	0.908	0.904
0.40	0.908	0.904	0.902
0.80	0.853	0.867	0.891
0.90	0.874	0.849	0.897
0.95	0.855	0.853	0.900
0.975	0.851	0.835	0.899
0.99	0.816	0.837	0.918
1.00	0.822	0.838	0.884

Eight-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.888	0.896	0.900
0.40	0.898	0.886	0.914
0.80	0.857	0.844	0.896
0.90	0.827	0.828	0.881
0.95	0.802	0.818	0.890
0.975	0.791	0.786	0.882
0.99	0.775	0.794	0.899
1.00	0.773	0.767	0.862

Ten-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.917	0.889	0.914
0.40	0.888	0.891	0.898
0.80	0.843	0.857	0.903
0.90	0.820	0.831	0.882
0.95	0.800	0.800	0.886
0.975	0.786	0.777	0.869
0.99	0.739	0.776	0.893
1.00	0.752	0.768	0.836

Table 1.3. Simulation results of the probability of coverage using $APV_2(2)$

One-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.907	0.891	0.899
0.40	0.897	0.904	0.908
0.80	0.897	0.891	0.904
0.90	0.892	0.902	0.896
0.95	0.909	0.894	0.911
0.975	0.906	0.878	0.927
0.99	0.898	0.872	0.932
1.00	0.894	0.881	0.904

Two-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.896	0.906	0.888
0.40	0.900	0.899	0.912
0.80	0.887	0.893	0.912
0.90	0.857	0.887	0.911
0.95	0.899	0.884	0.919
0.975	0.888	0.867	0.920
0.99	0.877	0.877	0.940
1.00	0.861	0.878	0.910

Four-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.897	0.912	0.904
0.40	0.911	0.905	0.904
0.80	0.860	0.872	0.896
0.90	0.879	0.859	0.909
0.95	0.862	0.865	0.924
0.975	0.857	0.844	0.926
0.99	0.826	0.849	0.948
1.00	0.832	0.846	0.923

Eight-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.893	0.899	0.901
0.40	0.898	0.887	0.915
0.80	0.861	0.849	0.900
0.90	0.833	0.836	0.907
0.95	0.816	0.822	0.921
0.975	0.807	0.796	0.914
0.99	0.791	0.809	0.929
1.00	0.789	0.774	0.901

Ten-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.918	0.893	0.914
0.40	0.889	0.891	0.899
0.80	0.847	0.857	0.905
0.90	0.823	0.836	0.896
0.95	0.812	0.810	0.921
0.975	0.797	0.782	0.910
0.99	0.754	0.787	0.927
1.00	0.763	0.776	0.881

Table 1.4. Bootstrap results of the probability of coverage using $APV_2(1)$

One-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.859	0.861	0.858
0.40	0.791	0.785	0.811
0.80	0.794	0.790	0.825
0.90	0.815	0.802	0.868
0.95	0.817	0.803	0.853
0.975	0.807	0.817	0.902
0.99	0.786	0.834	0.894
1.00	0.786	0.826	0.887

Two-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.895	0.892	0.905
0.40	0.848	0.859	0.863
0.80	0.815	0.809	0.836
0.90	0.811	0.808	0.862
0.95	0.829	0.805	0.877
0.975	0.803	0.811	0.891
0.99	0.787	0.833	0.891
1.00	0.782	0.842	0.891

Four-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.878	0.892	0.892
0.40	0.880	0.869	0.881
0.80	0.831	0.814	0.852
0.90	0.823	0.802	0.836
0.95	0.813	0.800	0.855
0.975	0.781	0.814	0.893
0.99	0.771	0.814	0.897
1.00	0.791	0.811	0.879

Eight-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.897	0.894	0.890
0.40	0.880	0.875	0.896
0.80	0.852	0.834	0.868
0.90	0.797	0.799	0.873
0.95	0.764	0.778	0.874
0.975	0.742	0.777	0.884
0.99	0.725	0.786	0.860
1.00	0.751	0.785	0.874

Ten-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.891	0.899	0.905
0.40	0.895	0.882	0.902
0.80	0.837	0.823	0.864
0.90	0.785	0.777	0.852
0.95	0.753	0.749	0.882
0.975	0.714	0.741	0.876
0.99	0.693	0.745	0.860
1.00	0.714	0.747	0.872

Table 1.5. Bootstrap results of the probability of coverage using $APV_2(2)$

One-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.863	0.866	0.861
0.40	0.792	0.791	0.811
0.80	0.796	0.793	0.829
0.90	0.817	0.803	0.871
0.95	0.7821	0.804	0.860
0.975	0.809	0.821	0.918
0.99	0.791	0.838	0.903
1.00	0.789	0.831	0.900

Two-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.898	0.894	0.907
0.40	0.850	0.859	0.866
0.80	0.817	0.814	0.841
0.90	0.815	0.813	0.871
0.95	0.829	0.811	0.890
0.975	0.808	0.816	0.910
0.99	0.793	0.840	0.909
1.00	0.787	0.848	0.914

Four-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.879	0.894	0.892
0.40	0.893	0.870	0.881
0.80	0.834	0.821	0.855
0.90	0.811	0.805	0.859
0.95	0.812	0.804	0.894
0.975	0.807	0.823	0.926
0.99	0.791	0.826	0.932
1.00	0.803	0.820	0.911

Eight-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.900	0.895	0.893
0.40	0.883	0.879	0.897
0.80	0.856	0.835	0.874
0.90	0.801	0.807	0.900
0.95	0.770	0.784	0.914
0.975	0.752	0.788	0.931
0.99	0.732	0.798	0.924
1.00	0.763	0.802	0.933

Ten-Step Ahead:

Rho	PW-OLS2	PW-WSLS	PW-MWSLS
0.00	0.891	0.900	0.906
0.40	0.897	0.883	0.903
0.80	0.838	0.824	0.869
0.90	0.789	0.784	0.871
0.95	0.762	0.754	0.915
0.975	0.719	0.756	0.928
0.99	0.703	0.759	0.919
1.00	0.728	0.762	0.933

Table 1.6. Autocorrelations and partial autocorrelations of the series $\{\hat{u}_t\}$

	Order	Country					
		Canada	France	Germany	Italy	U.K.	U.S.
ACF	1	0.9792	0.9274	0.9357	0.9365	0.9500	0.9814
	2	0.9534	0.8505	0.8848	0.8453	0.9008	0.9624
	3	0.9253	0.7570	0.8356	0.7402	0.8431	0.9435
	4	0.8929	0.6660	0.7934	0.6374	0.7684	0.9241
	5	0.8608	0.5743	0.7196	0.5486	0.6964	0.9035
	6	0.8292	0.4901	0.6563	0.4828	0.6174	0.8831
	7	0.7980	0.4141	0.5978	0.4282	0.5308	0.8632
	8	0.7671	0.3427	0.5469	0.3848	0.4550	0.8438
	9	0.7373	0.2803	0.4959	0.3487	0.3944	0.8250
	10	0.7062	0.2159	0.4525	0.3004	0.3371	0.8068
	11	0.6721	0.1507	0.4163	0.2410	0.2859	0.7885
	12	0.6386	0.0862	0.3760	0.1690	0.2446	0.7699
PACF	1	0.9792	0.9274	0.9357	0.9365	0.9500	0.9814
	2	-0.1340	-0.0690	0.0744	-0.2589	-0.0167	-0.0211
	3	-0.0552	-0.1593	-0.0022	-0.1215	-0.1136	-0.0053
	4	-0.1109	-0.0303	0.0335	-0.0039	-0.2103	-0.0248
	5	0.0178	-0.0483	-0.2679	0.0528	-0.0147	-0.0409
	6	-0.0031	-0.0066	-0.0011	0.0938	-0.0886	-0.0055
	7	-0.0013	0.0012	0.0053	-0.0335	-0.1119	0.0022
	8	-0.0196	-0.0331	0.0148	0.0088	0.0538	0.0051
	9	0.0088	0.0002	0.0457	0.0072	0.1531	0.0049
	10	-0.0598	-0.0733	0.0142	-0.1460	0.0103	0.0063
	11	-0.0863	-0.0698	0.0307	-0.0794	-0.0392	-0.0174
	12	0.0057	-0.0439	-0.0692	-0.0948	0.0238	-0.0166
Ljung-Box Q-statistics	6	823.85 (0.00)	358.09 (0.00)	595.06 (0.00)	341.58 (0.00)	644.83 (0.00)	699.08 (0.00)
	12	1364.67 (0.00)	410.99 (0.00)	819.30 (0.00)	415.67 (0.00)	799.80 (0.00)	1007.86 (0.00)
	18	1647.14 (0.00)	417.55 (0.00)	876.46 (0.00)	419.89 (0.00)	825.25 (0.00)	1121.69 (0.00)
	24	1744.92 (0.00)	452.00 (0.00)	882.19 (0.00)	430.27 (0.00)	829.23 (0.00)	1146.30 (0.00)

Note: The numbers in the parenthesis are the p-value of the Ljung-Box Q-statistics.

Table 1.7. The AIC and SBC values for selecting AR models for error series $\{u_t\}$

Country	AIC (SBC) Values					
	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)	AR(6)
Canada	-646.46	-654.68	-654.37	-655.33*	-654.27	-653.48
	(-643.42)	(-648.59)**	(-645.24)	(-643.16)	(-639.05)	(-635.22)
France	-543.30	-551.31	-559.46*	-557.52	-556.22	-555.69
	(-540.68)	(-546.06)	(-551.59)**	(-547.02)	(-543.09)	(-539.94)
Germany	-473.84	-473.20	-471.95	-469.99	-488.29*	-486.66
	(-470.90)	(-467.32)	(-463.13)	(-458.22)	(-473.59)**	(-469.01)
Italy	-488.05	-520.63	-518.77	-520.93*	-518.96	-520.84
	(-485.45)	(-515.42)**	(-510.95)	(-510.51)	(-505.94)	(-505.21)
U.K.	-628.03	-626.07	-625.38	-629.91*	-628.07	-626.37
	(-624.99)**	(-619.98)	(-616.25)	(-617.73)	(-612.86)	(-608.11)
U.S.	-653.46	-666.37	-666.48*	-664.48	-662.51	-662.53
	(-650.43)	(-660.31)**	(-657.39)	(-652.36)	(-647.35)	(-644.35)

Note: * indicates the best model chosen by AIC, and ** indicates the best model chosen by SBC.

Table 1.8. Autocorrelations and partial autocorrelations of the series $\{\hat{\varepsilon}_t\}$

		Country					
	Order	Canada	France	Germany	Italy	U.K.	U.S.
ACF	1	0.0058	-0.0084	0.0165	0.0040	-0.0060	-0.0274
	2	0.0139	0.0123	0.0110	-0.0528	-0.0235	0.0635
	3	0.0148	0.0040	0.0332	0.0251	-0.0534	0.0069
	4	-0.0986	-0.0972	0.0364	0.1077	0.0271	0.0404
	5	0.0600	0.0616	-0.0548	-0.1543	0.0596	-0.1056
	6	-0.0209	-0.0523	-0.0240	-0.0394	0.1273	0.0137
	7	0.0384	0.0882	-0.0845	0.0221	-0.0870	0.0106
	8	-0.0436	-0.0599	-0.0881	-0.0071	-0.1269	-0.1277
	9	0.0809	-0.0086	-0.0162	0.0618	0.0487	0.0069
	10	0.1380	0.2424	0.0336	0.1159	0.0321	0.0869
	11	-0.0948	0.1067	0.0135	0.0869	-0.0294	0.0963
	12	0.0095	-0.0885	-0.0093	0.0091	-0.1420	-0.0499
PACF	1	0.0058	-0.0084	0.0165	0.0040	-0.0060	-0.0274
	2	0.0138	0.0122	0.0108	-0.0528	-0.0235	0.0628
	3	0.0146	0.0042	0.0328	0.0256	-0.0537	0.0103
	4	-0.0990	-0.0974	0.0353	0.1051	0.0259	0.0371
	5	0.0614	0.0605	-0.0567	-0.1548	0.0577	-0.1053
	6	-0.0197	-0.0498	-0.0242	-0.0270	0.1274	0.0037
	7	0.0407	0.0887	-0.0854	0.0040	-0.0807	0.0237
	8	-0.0567	-0.0709	-0.0838	-0.0157	-0.1212	-0.1292
	9	0.0961	0.0038	-0.0073	0.1013	0.0545	0.0072
	10	0.1300	0.2343	0.0405	0.1001	0.0130	0.0947
	11	-0.0920	0.1378	0.0235	0.0831	-0.0516	0.1061
	12	-0.0099	-0.1297	-0.0123	0.0213	-0.1453	-0.0471
Ljung-Box Q-statistics	6	2.321 (0.313)	1.814 (0.612)	0.944 (0.331)	4.397 (0.111)	3.940 (0.139)	2.908 (0.573)
	12	8.776 (0.362)	12.389 (0.192)	3.450 (0.841)	7.333 (0.501)	12.094 (0.147)	8.966 (0.535)
	18	16.272 (0.297)	13.934 (0.531)	10.185 (0.679)	11.120 (0.677)	22.859 (0.063)	14.817 (0.538)
	24	21.747 (0.354)	19.245 (0.569)	11.325 (0.912)	13.166 (0.870)	27.741 (0.116)	18.109 (0.699)

Note: The numbers in the parenthesis are the p-value of the Ljung-Box Q-statistics.

Table 1.9. Actual values, forecast values, and prediction intervals for the LGDP

Country		LGDP Actual	LGDP Forecast	$APV_2(1)$ PI	$APV_2(1)$ Length	$APV_2(2)$ PI	$APV_2(2)$ Length
Canada	1997:2	4.6542	4.6530	(4.6373, 4.6687)	0.0314	(4.6239, 4.6822)	0.0583
	1997:3	4.6684	4.6620	(4.6396, 4.6844)	0.0447	(4.6081, 4.7159)	0.1078
	1997:4	4.6763	4.6713	(4.6436, 4.6989)	0.0552	(4.5927, 4.7498)	0.1572
	1998:1	4.6832	4.6804	(4.6483, 4.7126)	0.0643	(4.5772, 4.7837)	0.2065
	1998:2	4.6858	4.6896	(4.6533, 4.7258)	0.0725	(4.5617, 4.8174)	0.2557
	1998:3	4.6922	4.6987	(4.6587, 4.7387)	0.0800	(4.5462, 4.8512)	0.3049
	1998:4	4.7038	4.7078	(4.6643, 4.7514)	0.0871	(4.5308, 4.8849)	0.3542
	1999:1	4.7139	4.7170	(4.6701, 4.7639)	0.0938	(4.5153, 4.9187)	0.4034
	1999:2	4.7215	4.7261	(4.6760, 4.7762)	0.1002	(4.4998, 4.9524)	0.4526
	1999:3	4.7330	4.7352	(4.6821, 4.7884)	0.1064	(4.4843, 4.9861)	0.5018
France	1997:1	4.6243	4.6239	(4.5495, 4.6983)	0.1488	(4.5441, 4.7037)	0.1596
	1997:2	4.6309	4.6250	(4.5471, 4.7029)	0.1558	(4.5289, 4.7210)	0.1921
	1997:3	4.6392	4.6368	(4.5534, 4.7201)	0.1667	(4.5187, 4.7549)	0.2362
	1997:4	4.6498	4.6459	(4.5556, 4.7362)	0.1807	(4.5028, 4.7890)	0.2863
	1998:1	4.6590	4.6386	(4.5401, 4.7371)	0.1969	(4.4689, 4.8083)	0.3395
	1998:2	4.6675	4.6298	(4.5224, 4.7372)	0.2149	(4.4326, 4.8270)	0.3943
	1998:3	4.6723	4.6444	(4.5274, 4.7614)	0.2340	(4.4194, 4.8695)	0.4501
	1998:4	4.6787	4.6686	(4.5416, 4.7956)	0.2540	(4.4154, 4.9218)	0.5064
	1999:1	4.6822	4.6616	(4.5243, 4.7989)	0.2746	(4.3802, 4.9430)	0.5628
	1999:2	4.6884	4.6306	(4.4827, 4.7784)	0.2956	(4.3209, 4.9402)	0.6192
Germany	1996:3	4.6124	4.6190	(4.5084, 4.7296)	0.2212	(4.5077, 4.7303)	0.2226
	1996:4	4.6177	4.6308	(4.5195, 4.7421)	0.2226	(4.5179, 4.7436)	0.2257
	1997:1	4.6206	4.6418	(4.5294, 4.7543)	0.2248	(4.5265, 4.7572)	0.2307
	1997:2	4.6307	4.6577	(4.5438, 4.7716)	0.2279	(4.5391, 4.7763)	0.2373
	1997:3	4.6357	4.5800	(4.4642, 4.6957)	0.2316	(4.4573, 4.7026)	0.2453
	1997:4	4.6408	4.6056	(4.4877, 4.7235)	0.2359	(4.4784, 4.7329)	0.2545
	1998:1	4.6538	4.6181	(4.4977, 4.7384)	0.2407	(4.4857, 4.7504)	0.2647
	1998:2	4.6536	4.6376	(4.5145, 4.7606)	0.2461	(4.4996, 4.7755)	0.2758
	1998:3	4.6624	4.6236	(4.4976, 4.7495)	0.2519	(4.4798, 4.7674)	0.2876
	1998:4	4.6586	4.6488	(4.5198, 4.7778)	0.2580	(4.4988, 4.7987)	0.3000
Italy	1996:3	4.6160	4.6121	(4.6002, 4.6240)	0.0238	(4.5967,4.6275)	0.0308
	1996:4	4.6123	4.6139	(4.5969, 4.6309)	0.0340	(4.5881, 4.6398)	0.0517
	1997:1	4.6111	4.6196	(4.5985, 4.6407)	0.0421	(4.5837, 4.6556)	0.0719
	1997:2	4.6273	4.6261	(4.6015, 4.6506)	0.0492	(4.5801, 4.6720)	0.0919
	1997:3	4.6344	4.6331	(4.6054, 4.6609)	0.0556	(4.5772, 4.6890)	0.1118
	1997:4	4.6409	4.6396	(4.6089, 4.6704)	0.0615	(4.5738, 4.7054)	0.1316
	1998:1	4.6362	4.6457	(4.6122, 4.6792)	0.0671	(4.5700, 4.7214)	0.1514
	1998:2	4.6411	4.6513	(4.6151, 4.6875)	0.0724	(4.5657, 4.7369)	0.1712
	1998:3	4.6472	4.6568	(4.6180, 4.6956)	0.0775	(4.5613, 4.7523)	0.1910
	1998:4	4.6427	4.6623	(4.6210, 4.7035)	0.0825	(4.5569, 4.7676)	0.2107

Table 1.9. (continued)

Country		LGDP Actual	LGDP Forecast	$APV_2(1)$ PI	$APV_2(1)$ Length	$APV_2(2)$ PI	$APV_2(2)$ Length
United Kingdom	1997:2	4.6609	4.6660	(4.5477, 4.7844)	0.2367	(4.5467, 4.7854)	0.2387
	1997:3	4.6708	4.6800	(4.5610, 4.7991)	0.2381	(4.5583, 4.8017)	0.2434
	1997:4	4.6754	4.6944	(4.5743, 4.8145)	0.2402	(4.5691, 4.8197)	0.2506
	1998:1	4.6808	4.7092	(4.5876, 4.8307)	0.2431	(4.5793, 4.8391)	0.2598
	1998:2	4.6855	4.7238	(4.6005, 4.8470)	0.2465	(4.5884, 4.8591)	0.2706
	1998:3	4.6908	4.7382	(4.6130, 4.8634)	0.2504	(4.5969, 4.8795)	0.2826
	1998:4	4.6913	4.7525	(4.6252, 4.8798)	0.2547	(4.6048, 4.9002)	0.2955
	1999:1	4.6938	4.7665	(4.6368, 4.8962)	0.2593	(4.6120, 4.9210)	0.3089
	1999:2	4.7011	4.7803	(4.6481, 4.9124)	0.2643	(4.6188, 4.9417)	0.3229
	1999:3	4.7088	4.7938	(4.6590, 4.9285)	0.2694	(4.6252, 4.9623)	0.3371
United States	1996:4	4.6530	4.6496	(4.6337, 4.6654)	0.0317	(4.6222, 4.6769)	0.0547
	1997:1	4.6633	4.6570	(4.6345, 4.6796)	0.0452	(4.6071, 4.7070)	0.0999
	1997:2	4.6730	4.6647	(4.6368, 4.6925)	0.0557	(4.5923, 4.7371)	0.1448
	1997:3	4.6833	4.6723	(4.6400, 4.7047)	0.0647	(4.5776, 4.7671)	0.1896
	1997:4	4.6906	4.6800	(4.6437, 4.7164)	0.0727	(4.5629, 4.7972)	0.2343
	1998:1	4.7041	4.6877	(4.6476, 4.7278)	0.0802	(4.5482, 4.8272)	0.2790
	1998:2	4.7086	4.6954	(4.6519, 4.7390)	0.0871	(4.5336, 4.8573)	0.3237
	1998:3	4.7176	4.7031	(4.6563, 4.7499)	0.0937	(4.5189, 4.8873)	0.3684
	1998:4	4.7322	4.7108	(4.6608, 4.7608)	0.0999	(4.5043, 4.9174)	0.4131
	1999:1	4.7428	4.7185	(4.6655, 4.7715)	0.1059	(4.4896, 4.9474)	0.4578

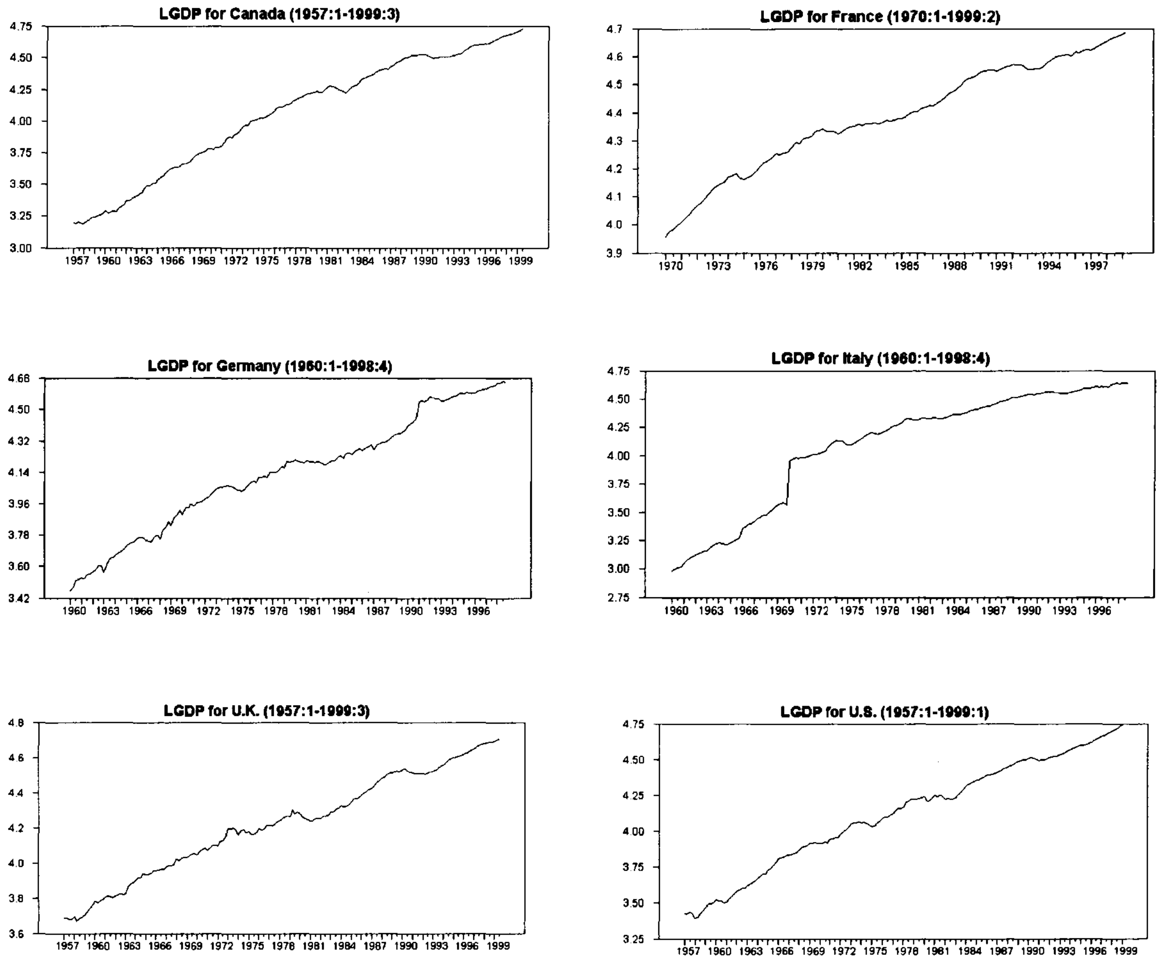

Figure 1.1. The logarithm of the quarterly RGDP series

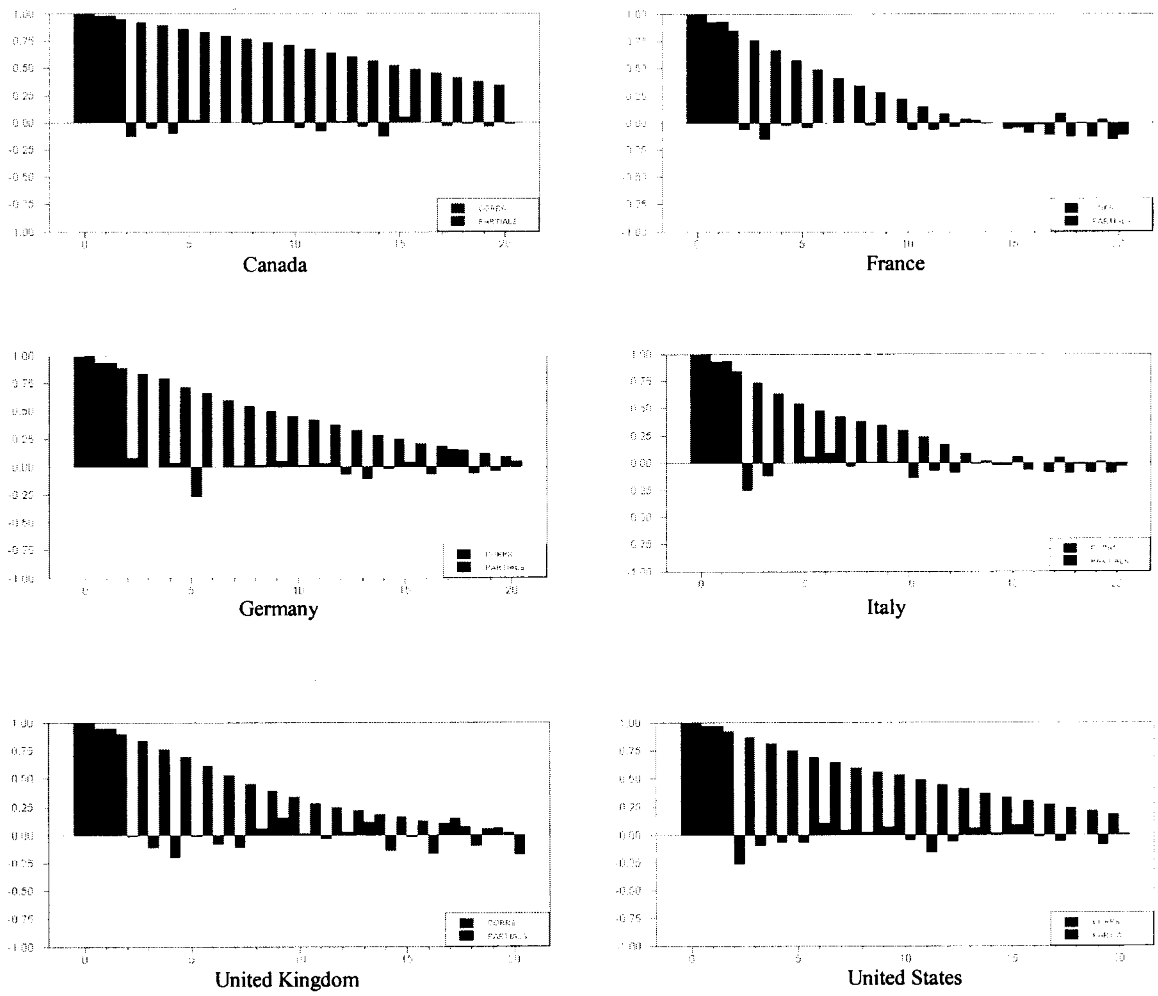

Figure 1.2. The sample ACF and PACF of the series $\{\hat{u}_t\}$

CHAPTER II.

INFLATION EXPECTATION RECONSIDERED

1. Introduction

It is widely acknowledged that inflation will not only affect individuals but also cause problems for the whole economy. Inflation can affect individuals through the erosion of real income and purchasing power. Its uncertainty may keep the public from spending and keep firms from investing. Inflation also will affect a country's imports and exports if domestic prices increase faster than those in other countries, thus having a negative effect on the balance of payments. The central bank's main task is to achieve and maintain price stability, or keep inflation low and stable, which is a prerequisite for sustainable economic development over the longer term.

To achieve the objective of price stability, central banks use intermediate objectives for monetary policy, such as the money supply, the exchange rate, the level of interest rates, or the volume of credit extended by banking institutions. Since the early 1990s, a number of central banks (for example, New Zealand, Canada, and the United Kingdom) introduced a new monetary policy strategy based on the direct targeting of a particular measure of inflation. The targets are expressed either as a range for inflation over time, or as a path for the inflation rate (Kahn & Parrish, 1998). By focusing on inflation targeting, monetary policy helps to moderate fluctuations in employment and domestic output.

Although the U.S. Fed has not formally adopted inflation-targeting, Clarida, Gali, and Gertler (2000) showed that the Fed's conduct of policy was consistent with a version of inflation-targeting called the Taylor Principle during the 1980s and 1990s, a period when inflation in the U.S. was reduced substantially and subsequently was maintained at a low, stable level. The Taylor Principle (Taylor, 1993) says that a central bank should focus on hitting a chosen target for the inflation rate, so long as output is not too far from the level consistent with the natural rate of unemployment. To do so, the central bank first chooses a target for the long-run nominal interest rate based on its inflation target. When the expected rate of inflation rises, the central bank should act aggressively by raising the nominal interest rate above its long-run target by more than the increase in expected inflation, which results in an increase in the real interest rate, which in turn reduces demand and contains the inflation. In short, the central bank will be successful in balancing its concerns about inflation and output if it reacts to expected inflation with an increase in the nominal interest rate that is larger than the increase in expected inflation. Then the problem comes down to the measurement of expected inflation or of indicators of expected inflation.

Moreover, getting good estimated time series of inflation expectations is important due to the role played by expected rate of inflation in various economic theories: models of wage and price determination, the Fisher effect on nominal interest rates, the Lucas supply curve, the Phillips curve, etc. It is important particularly because expected inflation is highly correlated with the time path of real output and inflation. For example, the most convincing explanations of the coexistence of high and increasing unemployment with rapid, accelerating inflation contain an expectations hypothesis that prices rise partly due to people expecting them to rise. In empirical studies, there has been a trend toward using more direct measures of inflation expectation rather than using simple proxies.

Hamilton (1992) noted that commodity futures prices often lead to revisions of the inference about aggregate price expectations, and proposed a new procedure incorporating the commodity futures prices to construct the inflation expectations taking into account not only the information available at the time of people's forecast, but also the information inferred by econometricians after the fact, in an effort to estimate the people's true inflation expectations. We adopt the innovative methodology of Hamilton to construct an expectation of inflation series, which gives a different inflation expectation measure that may help us to better understand the economic theories related with inflation expectation. Furthermore, the inflation expectation constructed using such methodology can be decomposed into anticipated and unanticipated parts, which can help us distinguish between different macroeconomic theories that explain the linkage between the nominal disturbance and real economic activity, and assess the credibility of the Fed's monetary policy.

In Section 2, we review the literature on different approaches to measuring inflation expectations, hypotheses of expectation formation, and the theoretical and empirical studies in which expected inflation plays an important role. Section 3 introduces the underlying theory of the methodology and the data we will use in this paper to form the expectation of inflation. Section 4 presents the empirical results, and Section 5 provides the comparison of the inflation expectations constructed from the model presented in this paper and the inflation expectations forecasted from the simple univariate time series model and the interest rate model. The conclusions and possible future research topics are discussed in the final section, section 6.

2. Literature Review

2.1. Theoretical and Empirical Studies Using Inflation Expectations

Inflation expectations play an important role in some key economic theories, such as the Fisher effect hypothesis and the Phillips curve. Empirically evaluating these theories can provide a better understanding of the economy and therefore may provide helpful information about the future conditions of the economy.

2.1.1. Fisher hypothesis

Irving Fisher's hypothesis about the impact of inflationary expectations on nominal interest rates has been studied extensively. Fisher (1930) claimed that with perfect foresight and a well-functioning capital market, there is a one-to-one relationship between inflation and nominal interest rates, with real interest rates unrelated to the expected rate of inflation and determined entirely by the real factors in an economy. However, with limited information, Fisher hypothesized that the expected real rate of interest r^e is the nominal rate i minus the expected rate of inflation π^e:

$$r^e = i - \pi^e \tag{2.1}$$

Equivalently, the Fisher hypothesis can be summarized in mathematical terms as:

$$i_t = E_t[\pi_t] + E_t[r_t] \tag{2.2}$$

where here and in the remainder of this paper, $E_t[\bullet]$ denotes the expectation conditional on period t information.

In the long run, the economy returns to full employment level of output, the real interest rate returns to its full-employment level r*, the actual inflation and expected inflation converge ($\pi^e = \pi$), providing the long run relationship among the nominal interest rate i, the inflation rate π, and the real interest rate r*: $r^* = i - \pi$.

In previous work, as pointed out by Mishkin (1992), examination of the Fisher effect involved testing for a significant correlation of the level of interest rates and the level of future inflation, i.e., testing for a significant coefficient for the term i_t^m in the regression equation:

$$\pi_t^m = \alpha_m + \beta_m i_t^m + \eta_t^m \tag{2.3}$$

where π_t^m is the m-period future inflation rate from time t to t + m; and i_t^m is the m-period interest rate known at time t. However, this regression did not make a distinction between short-run and long-run forecasting ability, hence did not distinguish the short-run Fisher effect (a change in the interest rate is associated with an immediate change in the expected inflation rate) from the long-run Fisher effect (the expected inflation rate will tend to be high when the interest rate is higher for a long period of time).

In addition, findings from this regression might be spurious if the short-term interest rates and future inflation rates had unit roots. In this case, the correct procedure to test the long-run Fisher effect is to test for cointegration between π_t^m *and* i_t^m in the regression (2.3). To look at it another way, this is to test for unit roots in the ex ante real interest rate under the assumption of rational expectations:

$$rr_t^m = i_t^m - E_t[\pi_t^m] \tag{2.4}$$

To test for a short-run Fisher effect, we should expect to find a significant positive coefficient β_m in the following regression:

$$E_t[\pi_t^m] - E_{t-1}[\pi_{t-1}^m] = \alpha_m + \beta_m[i_t^m - i_{t-1}^m] + u_t^m \tag{2.5}$$

Empirical studies give inconsistent conclusions about the presence of the Fisher effect. For example, supporting evidence is given by Fama (1975), Atkins (1989), Bonham (1991) and Wallace and Warner (1993), while Hess and Bicksler (1975), Carlson (1977), and Inder and Silvapulle (1993) found no evidence of the Fisher relationship[1]. Therefore, alternative and more accurate estimates of inflation expectation may be helpful in resolving the puzzle.

2.1.2. Phillips curve

The Phillips Curve describes an empirical relationship between price or wage inflation and the unemployment rate: the higher the rate of unemployment, the lower the rate of inflation, suggesting a trade-off between inflation and unemployment. The Phillips Curve did well in the 1950s and 1960s, but it failed to explain a period of coexistence of high unemployment and high inflation in the 1970s.

To explain such a change, Friedman (1968) and Phelps (1967) pointed out that the original Phillips Curve failed to take into account the effects of expected inflation on wage setting, and proposed an expectation-augmented Phillips Curve. The Friedman-Phelps theory suggested that the inflation adjustment compensated for expected inflation, so the actual inflation was determined by both expected inflation and unemployment (or the level of output):

$$\pi = \pi^e + \lambda(Y - Y^*) \qquad (2.6)$$

where Y is the level of output, Y^* is the full-employment level of output, and π^e is the growth rate of wages that represents inflation adjustment or expected inflation. This modified Phillips Curve explains why prices may be rising even when unemployment is high if expected inflation is sufficiently high.

So the traditional Phillips Curve emphasized some output gap measure as the relevant indicator of real economic activity, while under certain conditions, as shown by Gali and Gertler (2000), the Phillips Curve could be explained as the relationship between marginal cost and the output gap with:

$$\pi_t = \lambda k x_t + \beta E_t[\pi_{t+1}] \qquad (2.7)$$

where $x_t \equiv Y_t - Y_t^*$, k is the output elasticity of marginal cost, and the product kx_t represents marginal cost at time t. Therefore, the new Phillips Curve states that inflation depends positively on the output gap and a cost-push term reflecting the influence of expected inflation.

A number of researchers have also proposed a hybrid version of the Phillips Curve, which is to let inflation depend on a convex combination of expected future inflation and lagged inflation, with the lagged inflation term helping to capture inflation persistence:

$$\pi_t = \delta x_t + (1 - \phi)E_t[\pi_{t+1}] + \phi\pi_{t-1} \qquad (2.8)$$

Despite the efforts researchers have put into explaining the relationship between inflation and unemployment, the results are inconclusive on whether the Phillips curve is linear or non-linear, and whether there is asymmetry present in the inflation-unemployment trade-off. Improved estimates of expected inflation might help us better understand the Phillips Curve.

2.1.3. Inflation expectation and monetary policy

Clarida, Gali, and Gertler (2000) studied the role of expected inflation in the monetary policy rule. They considered a simple baseline policy rule in which the federal funds rate was the instrument of monetary policy. The policy rule called for adjustment of the funds rate to the gaps between expected inflation and output and their respective target levels:

$$r_t^* = r^* + \beta(E_t[\pi_{t,k}] - \pi^*) + \gamma E_t[x_{t,q}] \tag{2.9}$$

where r_t^* is the target rate for the nominal federal funds rate in period t; r^* is the desired nominal rate when both inflation and output are at their target levels; $\pi_{t,k}$ denotes the percentage change in the price level between periods t and t+k; π^* is the target for inflation; $x_{t,q}$ is a measure of the average output gap between period t and t+q. This forward looking specification of the policy rule was assumed to provide a reasonably good description of the way major central banks around the world behave. Such a specification also allowed the central bank to consider a broad array of information in forming beliefs about the future conditions of the economy.

Attempts to explain the linkage between real economic activity and nominal disturbances fall into two classes of competing macroeconomic theories: the Keynesian-style models by Fischer (1977) and Taylor (1980) that stress the impact of anticipated monetary policy on real economic activity, and the neoclassical monetary business cycle models of Lucas (1973) or Sargent and Wallace (1975) that emphasize the effect of unanticipated changes in monetary policy on real economic activity. The disinflation that occurred in the early 1980's is widely noticed and used by economists to shed light on the competing economic theories. Such experience, as pointed out by Dotsey and DeVaro (1995), was useful to discriminate different sets of theories if the disinflation was largely anticipated implying that the Fed's monetary policy was credible. They applied the method proposed by Hamilton (1992) to decompose the public's expected inflation into anticipated inflation and unanticipated inflation over the period of 1970 to 1986, and concluded that the disinflation was largely unanticipated and that the Fed lacked credibility.

2.2. Measuring Expected Inflation

One way to obtain an estimate of the expected inflation rate is by direct measures derived from data generated by surveys. There are two types of surveys as pointed out by Smith (1982). The Livingston surveys for the United States ask for the exact figure expected for the value of a specific price index at some future point in time, which can be translated easily into expected inflation. In contrast, the Gallup surveys in the United Kingdom ask respondents to indicate the expected direction of inflation change.

2.2.1. Gallup surveys

Carlson and Parkin (1975) developed a technique for quantifying the results of the four-category Gallup 1961 survey: go up, go down, stay the same, and don't know. Based on this technique, they obtained a time series of the expected rate of inflation in the United Kingdom. In particular, they showed

$$\pi_t^e = -\delta_t\left(\frac{a_t + b_t}{a_t - b_t}\right) \tag{2.10}$$

where π_t^e is the expectation formed in period t of inflation for the next period t+1, δ_t is a scaling factor, a_t refers to the proportions of respondents expecting prices not to rise, while b_t refers to the proportions of respondents expecting prices to fall. They concluded that expected inflation might be viewed as being generated by a second-order error-learning process when the inflation rate was high:

$$\pi_t^e - \pi_{t-1}^e = \lambda_1(\pi_t - \pi_{t-1}^e) + \lambda_2(\pi_{t-1} - \pi_{t-2}^e) \tag{2.11}$$

where π_t is the actual rate of inflation at time t. And the inflation expectations could be approximated by a purely autoregressive model when inflation was mild. They also found that devaluation had a dramatic impact on expected inflation rate, and variables such as wage-price guidelines, indirect tax changes, and political factors had no significant effect on inflation expectations.

Smith (1982) extended the Carlson-Parkin (C-P) time series of consumers' inflation expectations over the period 1974-1977. He revised the series by using a different scaling factor and a different procedure to handle the "don't know" responses. The newly tabulated inflation expectations moved together with C-P inflation expectations, but had smaller values and less variation in general. Smith (1982) also classified the hypotheses of expectation formation into two basic groups: error-learning and extrapolative models.

First-order error-learning or adaptive expectations postulates that expectations are revised by a constant proportion of the most recent errors:

$$\pi_t^e - \pi_{t-1}^e = \lambda(\pi_t - \pi_{t-1}^e) \qquad 0 < \lambda \leq 1 \tag{2.12}$$

Frenkel (1975) outlined an error-learning inflation expectation formation model based on the distinction between long-term and short-term expectations. Long-term expectations were related to actual inflations by a first-order error-learning process:

$$\pi_t^{le} - \pi_{t-1}^{le} = \lambda(\pi_t - \pi_{t-1}^{le}) \tag{2.13}$$

where π_t^{le} is the long-term expectation formed at time t. The short-term expectations were revised by constant proportions of the most recent errors in short-term expectations and the difference between the long-term expectation and the actual rate of inflation:

$$\pi_t^e - \pi_{t-1}^e = \lambda_2(\pi_{t-1}^e - \pi_{t-2}^e) + \lambda_3(\pi_t^{le} - \pi_t) \tag{2.14}$$

If the long-term expectations were expressed as an infinite distributed lag of past actual inflation rates before substituting for π_t^{le}, then (2.13) and (2.14) implied:

$$\begin{aligned}\pi_t^e - \pi_{t-1}^e = (\lambda_2 - \lambda_3)(\pi_t - \pi_{t-1}) + \lambda_1\lambda_2(\pi_{t-1} - \pi_{t-2}^e) \\ + (1 - \lambda_1 - \lambda_2)(\pi_{t-1}^e - \pi_{t-2}^e)\end{aligned} \tag{2.15}$$

The second group of basic models, extrapolative expectations, hypothesized that expected inflation was given by the most recent actual rate of inflation adjusted by a constant proportion of the recent changes in the inflation rate:

$$\pi_t^e = \pi_t + \theta(\pi_t - \pi_{t-1}) \tag{2.16}$$

Smith (1982) encompassed both the error-learning model and the extrapolative model, and presented a general formulation for inflation expectations. The distributed lag of actual inflation and of previous expectations of inflation gave the expected inflation:

$$\begin{aligned}\pi_t^e = \mu_1\pi_t + \mu_2\pi_{t-1} + \cdots + \mu_{n+1}\pi_{t-n} \\ + \nu_1\pi_{t-1}^e + \nu_2\pi_{t-2}^e + \cdots + \nu_m\pi_{t-m}^e\end{aligned} \tag{2.17}$$

With $m = n+1 = 2$, $\mu_1 = 1-\nu_1 = \lambda_1$ and $\mu_2 = -\nu_2 = \lambda_3$, we get second-order error-learning. If $m = n+1 = 1$, and $\mu_1 = 1-\nu_1 = \lambda_1$, then the adaptive expectation model is obtained. If all the ν_i are zero, we get the general extrapolative model. If all the μ_i are zeros, expectation formation is an autoregressive process.

Batchelor and Orr (1988) used a richer set of surveys (the Gallup 1961, 1974, and 1981 surveys, and the EEC 1984 survey) to update the C-P inflation expectation series. This revision changed the estimated profile of consumer inflation expectations over time such that the new series was less volatile and more accurate than the original C-P series.

2.2.2. Livingston surveys

The Livingston survey was started in 1946 by the columnist Joseph Livingston. The Philadelphia Federal Reserve Bank took over the survey in 1990. The survey is a semi-annual survey that summarizes the forecasts of economists in business, industry, banking, academics, and government. It includes a variety of economic variables, of which only the price expectations has been studied extensively.

Turnovsky (1970) analyzed the Livingston Survey's inflation forecasts from 1954 to 1969. He performed bias tests on the survey data and found large inflation forecast errors in the late 1950s, especially strong under-predicted forecast during 1956-1958. The reported predictions were substantially more accurate for the 1960's than for the earlier period. One of the reasons he offered for this finding was that people did not have much incentive to forecast inflation during the 1950s because inflation was low on average. The increased inflation during the 1960's induced more thorough forecasting. He also tested a number of models explaining the formation of price expectations. The evidence showed that the extrapolative expectation hypothesis was the most satisfactory one from the point of view of goodness of fit.

In a 1975 article, Pesando evaluated the Livingston forecasts and found the one-period forecast to be downward-biased, which imparted a corresponding bias to forecasts of the 12-month rate of inflation. He suggested that perhaps the survey was not representative of people's true forecasts. If the survey did represent true forecasts, then people were not rational according to his tests. Carlson (1977) argued that the published data were flawed because they might have been adjusted judgmentally by Livingston, so he presented a new series of expected inflation rates based on the averages of the original responses to the

Livingston surveys, which was used to reject Fama's hypothesis that the short-term interest rate was a good predictor of inflation rate.

Figlewski and Wachtel (1981) analyzed the set of individual responses to the Livingston survey rather than the average forecasts across individuals used in the previous work. They found evidence of bias and inefficiency in the individual forecasts. Their evidence strongly indicated that a single-coefficient, time-invariant model cannot be viewed as an adequate representation of the complex inflation expectation process, and they found significant differences in expectations formation among individuals over time.

The above discussion presents the literature on the formation of inflation expectation from data generated by two surveys: the Gallup Survey and the Livingston Survey. Other surveys available to the public include, for example, the Survey of Professional Forecasters, Blue Chip Economic Indicators, and the National Association of Business Economists' Outlook. We will not introduce them in this paper since they are not widely used in the literature.

2.2.3. Time series model

As stated earlier, some researchers have investigated the measurement of expected inflation using direct observations on inflation forecasts from surveys, but there exists suspicion that the survey data violate the rationality criterion, namely, the rational expectations are optimal forecasts conditional on available information. One way to obtain rational expectations of inflation is to formulate and estimate a time series model and confine the relevant information set to the past values of the inflation rate. The Autoregressive Integrated

Moving Average (ARIMA) model is frequently used for this purpose. The general formula for the ARIMA(p, d, q) model of the inflation rate π_t is:

$$\nabla^d \pi_t = u + \sum_{i=1}^{p} \phi_i (\nabla^d \pi_{t-i} - u) + \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} \tag{2.18}$$

where

$\nabla^d = (1-B)^d$, is the backward difference operator of order d, d = 0, 1, 2, …;

ϕ_i are the autoregressive (AR) process parameters, i = 1, 2, …, p;

θ_j are the moving average (MA) process parameters, j = 1, 2, …, q;

ε_t represents independent and identically $(0, \sigma^2)$-distributed error terms.

Once the appropriate model is selected, the model is estimated and then used to construct the inflation expectations. For an ARMA(p, q) process of inflation rate series, the forecasted inflation expectations are generated using the following formula from Hamilton (1994):

$$\hat{\pi}_{t+s|t} = \begin{cases} u + \phi_1(\hat{\pi}_{t+s-1|t} - u) + \phi_2(\hat{\pi}_{t+s-2|t} - u) + \cdots + \phi_p(\hat{\pi}_{t+s-p|t} - u) \\ \quad + \theta_s \hat{\varepsilon}_t + \theta_{s+1}\hat{\varepsilon}_{t-1} + \cdots + \theta_q \hat{\varepsilon}_{t+s-q} \qquad for\ s = 1,\ 2, \cdots,\ q \\ u + \phi_1(\hat{\pi}_{t+s-1|t} - u) + \phi_2(\hat{\pi}_{t+s-2|t} - u) + \cdots + \phi_p(\hat{\pi}_{t+s-p|t} - u) \\ \qquad\qquad\qquad\qquad\qquad\qquad for\ s = q+1, q+2, \cdots \end{cases} \tag{2.19}$$

where

$$\hat{\varepsilon}_t = \pi_t - \hat{\pi}_{t|t-1}$$
$$\hat{\pi}_{\tau|t} = \pi_\tau \quad for\ \tau \le t$$

This can be applied to the ARIMA(p, d, q) process by noting that $\nabla^d\pi_t$ is an ARMA(p, q) process.

Pearce (1979) restricted the relevant information set to the past values of the Consumer Price Index (CPI), then modeled the U.S. monthly CPI series for a base period (January 1947 to April 1959) as ARIMA(0, 2, 1) process using the time series techniques of Box and

Jenkins. The estimated model was used to forecast the CPI eight months and fourteen months ahead. Six more months of data were added, the model was re-estimated, and another pair of forecasts was computed. The procedure was repeated until the last forecast date he used. His results indicated that the time series model outperformed the Livingston survey data in terms of mean squared forecast error.

Hafer and Hein (1985) fit a first-order moving average model to the change in the GNP deflator inflation series of the United States over the sample period 1953-1969. This ARIMA(0, 1, 1) process of π_t was used to generate inflation expectations. They then compared three methods of inflation forecasting: a time series approach, an interest rate model, and the responses to the ASA-NBER survey of professional forecasters. The general conclusion was that the survey responses provided the most accurate ex ante forecasts of inflation, and the time series model forecasted worse than those derived from the interest rate model for 1970-1979 period but produced more accurate forecast for the 1980-1984 period. In a later study, Hafer and Hein (1990) applied univariate time series models to six countries (Belgium, Canada, England, France, Germany, and the United States) from 1967 to 1986, and concluded that the time series forecasts of inflation had equal or lower forecast errors and unbiased predictions more often than interest rate-based forecasts (which will be discussed below).

More recent work by Eberts and Maurer (2002) investigated the Germany inflation rate series, which was constructed from the monthly seasonally adjusted price index for the living standard of all private households in West Germany over 1963 to 1994. They applied an ARIMA(1, 0, 1) model, and their conclusion is similar to that of Hafer and Hein (1990), which is the superiority of the univariate time series model over the interest rate model in the in-sample forecasting.

2.2.4. Interest rate model

Fisher (1930) pointed out that with perfect foresight and a well-functioning capital market, the one-period nominal rate of interest is the equilibrium real return plus the fully anticipated rate of inflation. This hypothesized relationship between the interest rate and the inflation rate has been tested extensively. The general findings are that there exist relationships between the current interest rate and past rates of inflation in a world of uncertainty where foresight is imperfect. As an alternative to estimating inflation expectations from survey data, some researchers, for example, Sargent (1969) and Feldstein and Eckstein (1970), have followed Fisher (1930) by assuming that a distributed lag of past price changes was an adequate, observable proxy for inflation expectations.

The use of the Fisher equation to generate forecasts of inflation was suggested by Fama (1975). He specified the nominal interest rate as a linear function of the real interest rate and the inflation rate. More specifically, he assumed that the real interest rate was constant and estimated the following regression over 1953 to 1971:

$$\widetilde{\Delta}_t = \alpha_0 + \alpha_1 R_t + \widetilde{\varepsilon}_t \tag{2.20}$$

where R_t is the nominal Treasury bill rate; $\widetilde{\Delta}_t$ represents the expected value of inflation defined as the change in CPI, and the tilda means the variables are random. He concluded that assumption of constant real interest rate provided good inflation forecasts for the sample period. Nelson and Schwert (1977) showed that inflation forecasts from a univariate time series model were about as reliable as those from Fama's Treasury bill rate model.

Subsequent works rejected the assumption of a constant real interest rate, instead finding that the expected real interest rate behaved as a random walk. For example, Hess and Bicksler (1975) provided evidence that expected real returns on Treasury bills wander through time; Fama and Gibbons (1982) found that when expected real returns were assumed to follow a random walk, the Treasury bill rates presented good inflation forecasts for the entire sample period 1953-1977.

In 1984, Fama and Gibbons argued that if the real interest rate behaved as a random walk, then changes in the observed, ex-post real interest rates could be modeled as a simple moving average model and be written as:

$$R_{t-1} - \pi_t = E_{t-1}[r_{t-1}] + \varepsilon_t \tag{2.21}$$

where R_{t-1} represents the one-month Treasury bill rate observed at the end of month t-1, π_t is the inflation rate (the change in the natural log of the U.S. CPI), $E_{t-1}[r_{t-1}]$ is the expected real return for month t, and ε_t is the unexpected component of the real return. Then the changes in the real return for t and t-1 could be modeled as:

$$(R_{t-1} - \pi_t) - (R_{t-2} - \pi_{t-1}) = \Delta E_{t-1}[r_{t-1}] + \varepsilon_t - \varepsilon_{t-1} \tag{2.22}$$

where $\Delta E_{t-1}[r_{t-1}]$ is the change in the expected real return. If $E_{t-1}[r_{t-1}]$ was a random walk, and both $\Delta E_{t-1}[r_{t-1}]$ and ε_t were white noise, then the difference in real returns could be represented as an MA(1) process:

$$(R_{t-1} - \pi_t) - (R_{t-2} - \pi_{t-1}) = u_t + \theta\, u_{t-1} \tag{2.23}$$

Therefore, the inflation expectations could be derived by subtracting the ex ante forecast of the real interest rate from the nominal interest rate observed at the end of period t-1:

$$(R_{t-1} - \pi_t) = (R_{t-2} - \pi_{t-1}) + u_t + \theta\, u_{t-1} \tag{2.24}$$

Using the above approach, Fama and Gibbons (1984) concluded that the interest rate model provided slightly better monthly forecasts of inflation than a univariate time series model.

Eberts and Maurer (2002) noted the continuously significant positive autocorrelations in the observed German real interest rate series, and suggested it was natural to forecast real interest rates by an adequate ARIMA time series model. They applied an ARIMA(1,0,1) process, which was selected by the SBC, to the monthly real interest rate over 1962 to 1994. They also compared the interest rate model with the time series model and concluded the superiority of the time series model in the in-sample forecasting, but none of the models gave attractive out-of-sample inflation expectation performance.

Christiano (1989) put forth another interest model using quarterly yields on a three-month Treasury bill rate R_t as the interest rate covering the period 1960 to 1989 with the following specification:

$$\Delta\pi_t = \beta_0 \Delta R_t + \sum_{i=1}^{4} \beta_i \Delta\pi_{t-i} + \varepsilon_t \tag{2.25}$$

where ΔR_t is defined as R_{t-1}–R_{t-2}. Christinao's model was a competing model with the P* model that was the same as the above formula (2.18) except ΔR_t was replaced by natural log of the ratio $P_{t-1}{}^{*}/P_{t-1}$. Both models were outperformed by a time series model.

From the reviews presented above, inflation expectation is useful in theoretical and empirical settings. There are different approaches to obtain measures of inflation expectations, and the conclusions are inconsistent about which model better represents the inflation expectation process. To obtain alternative, hopefully more accurate, estimates of inflation expectation, we will employ a procedure in this paper that is proposed by Hamilton (1992), and later applied by Dotsey and DeVaro (1995), to construct the expected inflation series by using a vector dynamic system and incorporating information from the commodity futures market. The details are presented later in Section 3 and 4. The constructed inflation expectation series is compared to the series obtained from the univariate time series model and the interest rate model, with the results reported in Section 5.

3. Methodology

Hamilton (1992) examined the implications of futures markets price data for measuring expectations about future aggregate prices. Using a vector dynamic system to model the relationship between aggregate prices and futures prices of different commodities, he showed that observations on contemporaneous futures prices quite often lead to a revision of the inference about ex ante expectations of aggregate prices.

Hamilton argued that it is reasonable to treat the futures price of a commodity as the market's best guess as to the future spot price of that commodity. He modeled the relationship between commodity prices and aggregate prices and used the model to decompose aggregate price changes into anticipated and unanticipated components. He concluded that the public anticipated stable consumer prices during the first year of the Great Depression, meaning that the initial deflation was largely unanticipated. Later in the Great Depression, markets anticipated deflation, but not as severe as what actually occurred. Dotsey and DeVaro (1995) used the method Hamilton proposed to construct estimates of ex ante inflation expectations and used those data to study whether the disinflation of the early 1980s was anticipated.

Hamilton is not the only one who examined the role of commodity prices in providing signals about the future direction of the economy, especially inflation. Cody and Mills (1991) argued that the information in commodity prices should be used in formulating monetary policy, since commodities are traded in continuous auction markets and these prices provide instantaneous information about the state of the economy.

Let us start by looking at the theory underlying Hamilton's method of measuring expected inflation.

3.1. Relation between aggregate price level and commodity prices

We introduce the vector dynamic system representing the relationship between the aggregate prices and the commodity futures prices in Section 3.1.3, which links the simple forecasting regression model for aggregate price presented in Section 3.1.1, and the commodity price forecasting model shown in Section 3.1.2.

3.1.1. The aggregate price level

Let p_t denote the log of an aggregate price index and let x_t be a subset of the variables that are used to forecast p_t. Then a simple forecasting regression model is:

$$p_{t+1} = x_t'\delta + u_{t+1} \tag{2.26}$$

i.e.

$$p_{t+1} = E[p_{t+1}|x_t] + u_{t+1}$$

where u_{t+1} is the regression error and is uncorrelated with x_t.

The market's rational forecast, p^e_{t+1}, can be written as:

$$p^e_{t+1} = x_t'\delta + \alpha_t \tag{2.27}$$

i.e.

$$p^e_{t+1} = E[p_{t+1}|I_t], \quad x_t \subseteq I_t$$

where the rational expectation hypothesis says that α_t represents information people have in addition to x_t that is useful for forecasting p_{t+1}. It is assumed to be white noise, and to be uncorrelated with x_t. Therefore, people use two types of information: the information x_t that is known to everyone including the econometrician, and the information α_t that is unknown to the econometrician but can be inferred from the commodity futures market. And these two types of information are orthogonal to each other.

Let a_{t+1} be the market's forecasting error, which is from $p_{t+1} = x_t'\delta + \alpha_t + a_{t+1}$, and is assumed to be white noise and uncorrelated with α_t and x_t:

$$a_{t+1} = p_{t+1} - p^e_{t+1} \tag{2.28}$$

Combining the above information, and assuming the regression error u_{t+1} is white noise, we obtain the following expression:

$$u_{t+1} = \alpha_t + a_{t+1} \tag{2.29}$$

with

$$\mathrm{Var}(u_{t+1}) = E[u^2_{t+1}] = \sigma^2_\alpha + \sigma^2_a \tag{2.30}$$

where σ^2_a is the variance of people's true forecast error, and σ^2_α is the variance of the omitted information term α_t.

3.1.2. The commodity futures market

Let $S_{j,t}$ denote the spot price of commodity j purchased at date t, and let $F_{j,t}$ denote the price of a one-period ahead forward contract. At date t, the contract is signed but no money changes hands, and at date t+1, the commodity is delivered at the contract price $F_{j,t}$.

If risk-neutral speculators have access to this market, then the contract price $F_{j,t}$ equals the expectation formed by market participants at date t as to the value the spot price will take on at date t+1:

$$F_{j,t} = E_t[S_{j,t+1}] \tag{2.31}$$

Following Hamilton (1992), we assume that the distribution of the log of the spot price $s_{j,t+1}$ conditional on information available at date t is normally distributed:

$$s_{j,t+1} \equiv \log(S_{j,t+1}) \sim N(u_{j,t}, \sigma_j^2) \tag{2.32}$$

then $S_{j,t+1}$ has a lognormal distribution with the density:

$$f(S_j; u_j, \sigma_j^2) = \frac{1}{S_j\sqrt{2\pi\sigma_j^2}} \exp[-\frac{1}{2\sigma_j^2}(\log(S_j) - u_j)^2] I_{(0,\infty)}(S_j) \tag{2.33}$$

and

$$E_t[S_{j,t+1}] = \exp(u_{j,t} + \sigma_j^2/2) \tag{2.34}$$

Substituting (2.34) into (2.31) and taking logs,

$$\log F_{j,t} = u_{j,t} + \sigma_j^2/2$$

or rewrite as

$$f_{j,t} = E_t[s_{j,t+1}] - k_j \tag{2.35}$$

where $k_j \equiv -\sigma_j^2/2$, which is an unrestricted constant that can also incorporate a constant risk premium.

The efficient market hypothesis claims that the market's error in forecasting the j-th commodity price $w_{j,t+1}$ can be observed directly from $s_{j,t+1} - f_{j,t}$, we can write as:

$$s_{j,t+1} - f_{j,t} = k_j + w_{j,t+1} \tag{2.36}$$

Let $q_j^a a_{t+1}$ denote the projection of the commodity price forecast error $w_{j,t+1}$, on the aggregate price forecast error, a_{t+1}. Then

$$w_{j,t+1} = q_j^a a_{t+1} + e_{j,t+1} \tag{2.37}$$

where the projection error $e_{j,t+1}$ denotes unanticipated movements in the price of commodity j in period t, that is, uncorrelated with aggregate price movements during that period.

The covariance between the observed aggregate regression error u_{t+1} and the observed commodity forecasting error $w_{j,t+1}$ must be due to the market's true error in forecasting aggregate prices a_{t+1}. This covariance is:

$$\begin{aligned} Cov(u_{t+1}, w_{j,t+1}) &= E[u_{t+1} w_{j,t+1}] \\ &= E[(\alpha_t + a_{t+1})(q_j^a a_{t+1} + e_{j,t+1})] \\ &= q_j^a \sigma_a^2 \end{aligned} \tag{2.38}$$

Next, let's consider the regression of the futures price $f_{j,t}$ on x_t:

$$f_{j,t} = x_t'\beta_j + v_{j,t} \tag{2.39}$$

where $v_{j,t}$ represents the information that market participants had beyond the information contained in x_t. It is assumed that sufficient lags of variables are included in x_t so that $v_{j,t}$ is white noise. Since $v_{j,t}$ is known to the market at time t, any correlation between $v_{j,t}$ and the aggregate regression error u_{t+1} must be due to α_t (the information people had at time t not contained in x_t). Let the projection of $v_{j,t}$ on α_t be:

$$v_{j,t} = q_j^\alpha \alpha_t + \varepsilon_{j,t} \tag{2.40}$$

where $\varepsilon_{j,t}$ is the information people had at time t about the future course of commodity j, that is, uncorrelated with aggregate price movements. The covariance between the observed aggregate regression error u_{t+1} and the observed commodity regression error $v_{j,t}$ is:

$$\begin{aligned} Cov(u_{t+1}, v_{j,t}) &= E[u_{t+1} v_{j,t}] \\ &= E[(\alpha_t + a_{t+1})(q_j^\alpha \alpha_t + \varepsilon_{j,t})] \\ &= q_j^\alpha \sigma_\alpha^2 \end{aligned} \tag{2.41}$$

3.1.3 Aggregate prices and n commodity prices

Define the n×1 vectors as the following:

$v_t \equiv (v_{1,t}, \dots, v_{n,t})'$

$w_{t+1} \equiv (w_{1,t+1}, \dots, w_{n,t+1})'$

$\varepsilon_{j,t} \equiv (\varepsilon_{1,t}, \dots, \varepsilon_{n,t})'$

$e_t \equiv (e_{1,t}, \dots, e_{n,t})'$

$q^\alpha \equiv (q_1^\alpha, \dots, q_n^\alpha)'$

$q^a \equiv (q_1^a, \dots, q_n^a)'$

And define the n×n matrices $\Sigma \equiv E[\varepsilon_t \varepsilon_t']$ and $S \equiv E[e_t e_t']$. Then from the preceding discussions we obtain the vector dynamic system, which consists of 2n+1 equations:

$$\begin{cases} p_{t+1} = x_t'\delta + u_{t+1} & \\ f_{j,t} = x_t'\beta + v_{j,t} & j = 1, \cdots, n \\ s_{j,t+1} - f_{j,t} = k_j + w_{j,t+1} & j = 1, \cdots, n \end{cases} \tag{2.42}$$

and the variance-covariance matrix of the observed error terms from these equations:

$$\Omega \equiv E\begin{bmatrix} u_{t+1} \\ v_t \\ w_{t+1} \end{bmatrix}[u_{s+1} \quad v_s' \quad w_{s+1}']$$

$$= \begin{cases} \begin{bmatrix} (\sigma_a^2 + \sigma_\alpha^2) & \sigma_\alpha^2 (q^\alpha)' & \sigma_a^2 (q^a)' \\ \sigma_\alpha^2 q^\alpha & [\sigma_\alpha^2 q^\alpha (q^\alpha)' + \Sigma] & 0 \\ \sigma_a^2 q^a & 0 & [\sigma_a^2 q^a (q^a)' + S] \end{bmatrix} & if \quad t = s \\ 0 & if \quad t \neq s \end{cases} \tag{2.43}$$

This vector dynamic system represents the relationship between the aggregate prices and the prices for j = 1, … , n different commodities.

The above dynamic system proposed by Hamilton (1992), as shown in (2.42) and (2.43), has two classes of restrictions:

(A). The only allowable explanatory variable in equation $s_{j,t+1} - f_{j,t} = k_j + w_{j,t+1}$ in (2.42) is a constant term k_j.

(B). The n×n block of (2.43) corresponding to the covariance between w_{t+1} and v_t' is forced to be zero.

The two restrictions reflect the risk neutral efficient market assumption that information available at time t (x_t or v_t) should be uncorrelated with the market's error in forecasting commodity prices (w_{t+1}). It is possible to form an inference about the market's expectations of aggregate prices without assuming risk-neutrality, in which case restrictions A and B can be relaxed to estimate the system.

Dotsey and DeVaro (1995) allowed for time-varying risk premiums in the case when w_{t+1} is correlated with time t information and is not normally distributed white noise. In that case, equation (2.36) is replaced by:

$$s_{j,t+1} - f_{j,t} = x_t'K + w_{j,t+1} \qquad j = 1, \ldots, n \tag{2.36'}$$

After the modification, $w_{j,t+1}$ in (2.36') is normally distributed white noise. They also modified (2.42) in the vector dynamic system, replacing p_{t+1} by Δp_{t+1}. So they estimate the following system of equations:

$$\begin{cases} \Delta p_{t+1} = x_t'\delta + u_{t+1} & \\ f_{j,t} = x_t'\beta + v_{j,t} & j = 1,\cdots,n \\ s_{j,t+1} - f_{j,t} = x_t'K + w_{j,t+1} & j = 1,\cdots,n \end{cases}$$

or, in matrix form:

$$\begin{bmatrix} \Delta p_{t+1} \\ f_t \\ s_{t+1} - f_t \end{bmatrix} = \begin{bmatrix} \delta' \\ \beta' \\ K' \end{bmatrix} x_t + \begin{bmatrix} u_{t+1} \\ v_t \\ w_{t+1} \end{bmatrix} \tag{2.42'}$$

plus the variance-covariance matrix of the observed error terms from above equations.

To examine the market expectations about price or to have an optimal inference, as suggested by Hamilton (1992), use not only the information people have available at the time of their forecasts, but also the information that the econometrician is able to observe only after the fact. The statistically optimal inference about people's ex ante expectation of price is given by $\hat{p}^e_{t+1}$, which is the estimate of p^e_{t+1}:

$$\begin{aligned} \hat{p}^e_{t+1} = x_t'\delta + [(q'S^{-1}q + \sigma_a^{-2})/\Delta][p_{t+1} - x_t'\delta] \\ + [q'\Sigma^{-1}/\Delta] \begin{bmatrix} f_{1,t} - x_t'\beta_1 \\ f_{2,t} - x_t'\beta_2 \\ f_{3,t} - x_t'\beta_3 \\ f_{4,t} - x_t'\beta_4 \end{bmatrix} - [q'S^{-1}/\Delta] \begin{bmatrix} s_{1,t+1} - f_{1,t} - k_1 \\ s_{2,t+1} - f_{2,t} - k_2 \\ s_{3,t+1} - f_{3,t} - k_3 \\ s_{4,t+1} - f_{4,t} - k_4 \end{bmatrix} \end{aligned} \tag{2.44}$$

where $\Delta = q'S^{-1}q + \sigma_a^{-2} + q'\Sigma^{-1}q + \sigma_\alpha^{-2}$. So, the econometrician's best ex-post estimate of the market's ex-ante forecast of the aggregate price level can be expressed as the sum of four terms: the first term is the simple inference $x_t'\delta$, the second term gives the weight to be placed on ex post values of the aggregate price level itself, the third term represents the weight on the commodity futures data, and the last term gives the weight to be placed on the market's errors in forecasting commodity prices.

Once we get the estimates for (2.42) and (2.43), we can analyze the weights of observations on ex post aggregate prices, ex post commodity forecast errors, and observations on contemporaneous futures prices on the optimal inference about people's ex ante expectations of price, then we can derive the ex ante expectations of inflation and continue with the analyses in which we are interested.

3.2. Estimation Methods

To identify the individual components of the variance-covariance matrix Ω, we need an additional restriction. Following Hamilton (1992), we assume $q_j^a = q_j^\alpha = q_j$ as restriction (C). Then the covariance between u_{t+1} and w_{t+1}' is proportional to the covariance between u_{t+1} and v_t':

$$E[u_{t+1}w_{j,t+1}]/E[u_{t+1}v_{j,t}] = \sigma_a^2/\sigma_\alpha^2 \qquad \text{for } j = 1, \dots, n \tag{2.45}$$

This additional restriction allows us to use the ratio of the covariance of u_{t+1} and w_{t+1} to the covariance of u_{t+1} and v_t to estimate $\sigma_a^2/\sigma_\alpha^2$. It also provides very useful information: a large covariance between u_{t+1} and $w_{j,t+1}$ can be taken as evidence that much of the residual u_{t+1} took people by surprise; a large covariance between u_{t+1} and $v_{j,t}$ suggests that little of the residual u_{t+1} took people by surprise. As pointed out by Dotsey and DeVaro (1995), restriction (2.45) is valid if anticipated and unanticipated movements affect s_{t+1} and p_{t+1} proportionately even though the absolute effects of an anticipated movement need not be the same as those of an unanticipated movement. For example, when the relevant aggregate information that agents possess is a demand shock that affects both the commodity prices and the aggregate prices in similar ways, then q_j^a should equal to q_j^α. This restriction, however, need not be valid, and can be tested with such restriction placed on one commodity when multiple commodities are used.

To estimate the dynamic system, we need to find a minimal set of explanatory variables x_t that assure u_{t+1} and $v_{j,t}$ are white noise. Hamilton (1992) chose x_t to include a constant, two seasonal dummies since all the variables have strong seasonal variations, two lags of prices, and spot commodity prices for corn, oats, and rye. So

$$x_t' = (1, d_{1t}, d_{2t}, p_t, p_{t-1}, s_{corn,t}, s_{oats,t}, s_{rye,t}) \tag{2.46}$$

Dotsey and DeVaro (1995) used almost the same variables, except they used soybeans commodity prices rather than rye prices due to data availability, and they used two lags of the inflation rate rather than two lags of the price level. So, in their system

$$x_t' = (1, d_{1t}, d_{2t}, \Delta p_t, \Delta p_{t-1}, s_{corn,t}, s_{oats,t}, s_{soybeans,t}) \tag{2.47}$$

The data on commodity prices are from annual reports of the Chicago Board of Trade. The logarithm of CPI is used as the aggregate price index p_t.

Then the system of equations in (2.42) and (2.43) is estimated by full-information maximum likelihood (FIML) subject to $q^a = q^\alpha = q$. And the log likelihood function is

$$l = \text{constant} - \frac{T}{2}\log|\Omega| - \frac{1}{2}\sum_{t=1}^{T}(y_t - Bx_t)'\Omega^{-1}(y_t - Bx_t) \tag{2.48}$$

where y_t is the vector of dependent variables, x_t is the vector of independent variables, B represents the vector of coefficients in (2.42), and Ω is the variance-covariance matrix defined in (2.43).

4. Empirical Results

4.1. Data

We use four commodities: corn, oats, soybean, and wheat, which are the only commodities that have observations available over our entire sample period from 1975 to 2001. The commodity prices are from the Chicago Board of Trade. The data are tri-annual data since the futures contract is four months in duration. We use I, II and III to represent the three observations each year. For example, 1980:I refers to the first observation in 1980, and 2000:III refers to the last observation in 2000 . The price of a futures contract that is about to expire is used as the spot price of that commodity, and the price of a four-month ahead future contracts at the same date is used as the futures price of that commodity. The aggregate price used is the monthly consumer price index (CPI) from *International Financial Statistics*. The observations for January, May, and September are picked to match the tri-annual frequency of commodity prices. The natural logarithm transformation is applied to all data. The graphs for these price series are shown in Figure 2.1. The inflation rate implied by the CPI is also graphed in Figure 2.1.

4.2. Estimation

Before we start estimating the system of equations, we perform Augmented Dickey-Fuller (ADF) tests on aggregate prices, commodity spot prices, and commodity futures prices over the period 1975:I to 2001:II.[2] The ADF test results are presented in Table 2.1. We can reject the null hypotheses of a unit root for all the series at the 5 percent significance level for the commodity spot price and commodity futures price of corn, oats, soybeans, and wheat. So these series are assumed to be stationary. For the aggregate price in levels, we fail to reject the unit root hypothesis, which is not surprising since the aggregate price is growing over the sample period. However, we are able to find that the aggregate price in differences, i.e., the inflation rate series, is stationary. We will use both the aggregate price in levels (Hamilton, 1992)[3] and the aggregate price in differences in our analyses, which are presented in Sections 4.2.1 and 4.2.2, respectively.

One of the hypotheses assumed in the model is the efficiency of the commodity markets, so the information available at time t should be uncorrelated with the market's error in forecasting commodity prices for time t+j. This is reflected in equation (2.36): $s_{j,t+1}-f_{j,t} = k_j+w_{j,t+1}$, which states that the error term $w_{j,t+1}$ is normally distributed white noise. We present the Ljung-Box Q-statistics in the first part of Table 2.2. The test results confirm that the white noise assumption is reasonable over our sample period. The skewness and kurtosis of each series suggest we cannot reject the assumption of normality. So equation (2.36) provides an adequate description of commodity market behavior.

4.2.1. Estimation using the aggregate price levels

As mentioned earlier, to estimate the dynamic system, we need to find a minimal set of explanatory variables x_t that assure u_{t+1} and $v_{j,t}$ are white noise. There is evidence in Table 2.3 that seasonal variations are correctly anticipated by the futures markets[4], so two seasonal dummies are included in the explanatory variables x_t. Therefore, we choose x_t to include a constant, two seasonal dummies, two lags of prices, and the spot commodity prices for corn, oats, soybean and wheat:

$$x_t' = (1, d_{1t}, d_{2t}, p_t, p_{t-1}, s_{c,t}, s_{o,t}, s_{s,t}, s_{w,t}) \tag{2.49}$$

Next, we examine whether u_{t+1} and $v_{j,t}$ are uncorrelated error series using the independent variables shown above. The results are presented in the second half of Table 2.2. For the estimated commodity regression error series $\{v_{j,t}\}$, we cannot reject the null hypotheses of no autocorrelation for corn, oats, and soybeans at the 5 percent significance level, but we show autocorrelation for wheat. For the estimated aggregate regression forecast error series $\{u_{t+1}\}$, we can say that it is white noise at the 3 percent significance level. The value of Ljung-Box Q-statistics in Table 2.4 shows that if we exclude wheat from x_t in (2.49), then the error series $\{w_{j,t+1}\}$ and $\{v_{j,t}\}$ are white noise at the 5 percent level, while $\{u_{t+1}\}$ is white noise at the 2 percent level. Therefore, we exclude wheat in our further analysis, and the explanatory variables in x_t are:

$$x_t' = (1, d_{1t}, d_{2t}, p_t, p_{t-1}, s_{c,t}, s_{o,t}, s_{s,t}) \qquad (2.49)'$$

So the system using aggregate price in levels is a seven-equation-system depicted by (2.42), and the variance-covariance matrix Ω of the observed error terms is shown in (2.43): f and s are 3×1 vectors containing the commodity futures prices and spot prices of corn, oats, and soybeans, respectively; the coefficient vector δ is a 8×1 vector, β is an 8×3 matrix, and κ is a 3×1 vector. The system is estimated with FIML, and the OPTIMUM procedure in the software GAUSS is used to conduct the analysis.

The Sims' (1980) adjusted likelihood-ratio test is used often to examine different specifications of nested models, such as the model with restrictions A, B and C against the specification that the model only has restrictions A and B. The test statistic is defined as $2(\frac{T-k}{T})(\mathit{likelihood}\,[\mathit{unrestricted}] - \mathit{likelihood}\,[\mathit{restricted}])$, which asymptotically has a Chi-square distribution with degrees of freedom equal to the difference between the number of parameters in the unrestricted model and the number of parameters in the restricted model. We use Sims adjusted likelihood-ratio test on our system with different restrictions imposed. The p-value for the test statistic of a joint likelihood-ratio test of the restricted model with restrictions A, B and C against the unrestricted model is 0.09. The p-value for the test of the model with restrictions A, B and C against the model with restrictions A and B is 0.85. And the p-value for the test of the model with restrictions A, B and C versus the model with restriction C only is 0.07. Taken together, these test results imply that we cannot reject the joint restrictions A, B and C at the 5 percent level. Therefore, the final system we estimate is the model using the aggregate price level, prices for the three commodities (corn, oats, and soybeans), and with restrictions A, B and C imposed. The estimation results using FIML are reported in Table 2.5.

The variance of the price forecast σ_u^2 is 0.43, about 0.13 is due to the variance of people's true forecast error σ_a^2, and about 0.30 comes from the variance of the omitted information σ_α^2. Note the system is estimated with restriction C in (2.45) imposed. Therefore large σ_α^2 implies the covariance between the aggregate regression error (u_{t+1}) and futures price regression error ($v_{j,t}$) is relatively large, or that little of the residual u_{t+1} took people by surprise over our sample period. So we conclude that the market apparently anticipated the change in the inflation over time.

From the FIML estimates in Table 2.5, the estimates of the predicted p_{t+1}^e are calculated using expression (2.44):

$$\begin{aligned}\hat{p}^e_{t+1} &= 1.5563 - 0.7277\,d_{1t} - 0.3096\,d_{2t} + 1.4750\,p_t - 0.4849\,p_{t-1} \\ &\quad - 0.0036\,s_{c,t} - 0.0065\,s_{o,t} + 0.0141\,s_{s,t} \\ &\quad + 0.6504\,(p_{t+1} - x_t'\delta) \\ &\quad - 0.0063\,(f_{c,t} - x_t'\beta_c) + 0.0045\,(f_{o,t} - x_t'\beta_o) + 0.0219\,(f_{s,t} - x_t'\beta_s) \\ &\quad + 0.0010(s_{c,t+1} - f_{c,t} - k_c) - 0.0007(s_{o,t+1} - f_{o,t} - k_o) \\ &\quad - 0.0013\,(s_{s,t+1} - f_{s,t} - k_s)\end{aligned} \tag{2.50}$$

The forecasts of aggregate prices are also decomposed into individual contributions from the simple regression forecast $x_t'\delta$, the ex post price term $p_{t+1}-x_t'\delta$, the future terms $f_{j,t}-x_t'\delta$, and the commodity market surprise term $s_{j,t+1}-f_{j,t}-k_j$. The actual value of the aggregate price and the contribution of each component to the inferred expectation of the aggregate price are displayed in Table 2.6. Observations on commodity futures prices are given relatively large weight in the optimal inference about people's ex ante expectations of aggregate price p^e_{t+1}, leading to inference revisions up to 31 basis points; while ex post commodity forecast errors are given relatively small weight, rarely amounting to more than 3 basis points, with the largest revision to the inference being about 6 basis points. Table 2.7 reports the one-period ahead actual inflation and expected inflation at annualized rates at the indicated time.

4.2.2. Application

The U.S. economy slowed down in late 1999 and 2000 after the longest period of post war economic expansion, entering a recession in March, 2001 according to the National Bureau of Economic Research. To boost aggregate demand and avert a recession, the Fed reduced its target for the federal funds rate by 50 basis points, to 6 percent, in January, 2001, and started an aggressive rate reduction cycle with seven consecutive rate cuts, bringing the rate down to 1.75 percent at the end of 2001, the lowest level in 40 years. Figure 2.2 shows that the Treasury bill rate decreased dramatically starting late 2000 and continued dropping through 2001, and the inflation rate was moderate and edging lower due to few indications of economic improvement in that year.

The actual inflation rate and the expected inflation rate for the period 1999 through 2001 are shown in Table 2.8. The expected rates of inflation are lower than the actual inflation rates in every four-month-ahead forecast, in seven out of eight cases over the eight-month forecast horizon and in two out of eight cases over the one-year forecast horizon. On average, the public expected 0.296 percent, 0.979 percent and 1.686 percent more deflation in the one-period ahead, two-period ahead, and three-period ahead forecasts, respectively. It is reasonable that starting from 1999, the public would have expected that the inflation rates would be lower four months later but still thought that the rates would go up in eight months or one year. But, as the economy slowed down, the Fed carried out its easing economy monetary policies, the public revised their expectation about the inflation rate, and believed that the inflation rates would be edging lower, so the public correctly anticipated the policy change. Thus the Fed's policy is credible over the recent economic downturn period.

4.2.3. Estimation using inflation rate

We also employ the inflation rate for the analysis of the system of (2.42) and (2.43) with p_t replaced by Δp_t, since the price level may be nonstationary. We start with the inclusion of a constant, two seasonal dummies, two lags of inflation, and the commodity spot prices for corn, oats, soybeans, and wheat in the vector of explanatory variables x_t.

The hypothesis of efficiency of the commodity market, i.e., that $w_{j,t+1}$ is normally distributed white noise, still holds since $w_{j,t+1}$ is exactly the same as in the previous case of using price levels. Next, we examine whether u_{t+1} and $v_{j,t}$ are white noise series. The results are presented in Table 2.9. The estimated aggregate regression forecast error series $\{u_{t+1}\}$ is white noise. For the estimated commodity regression errors $\{v_{j,t}\}$, we fail to reject the null hypotheses of no autocorrelation for corn and oats at the 1 percent significance level, but we fail to show no autocorrelation for soybeans and wheat. In further analysis, we exclude wheat but keep soybeans, as we did in section 4.2.1 in the seven-equation system.

The Sims' (1980) adjusted likelihood-ratio test is applied to examine different specifications of the model. The p-value for the test statistic of a joint likelihood-ratio test of the restricted model with restrictions A, B, and C against the unrestricted model is 0.15. The p-value for the test of the model with restrictions A, B, and C against the model with restrictions A and B is 0.36. And the p-value for the test of the model with restrictions A, B and C versus the model with restriction C only is 0.12. All test results imply that we cannot reject the joint restrictions A, B, and C at the 5 percent level. Therefore, the final system we estimate is the model using the inflation rate and three commodities (corn, oats, and soybeans) with restrictions A, B, and C imposed. The estimation results are reported in Table 2.10.

The variance of the inflation rate forecast σ_u^2 is 0.39, most of which is due to the variance of the public's forecast error $\sigma_a^2 = 0.27$. The rest comes from the variance of the omitted information σ_α^2. The relatively large covariance between the aggregate regression error (u_{t+1}) and the market's error in forecasting commodity j ($v_{j,t}$) implies that most of the residual u_{t+1} took people by surprise. This conclusion is different from what we found when we used the aggregate price level in the estimation. Then we concluded that little of the residual took people by surprise.

From the estimates of the FIML regression in Table 2.10, the estimates of the predicted Δp^{e}_{t+1} are calculated, and the forecasts of the inflation rate can be decomposed into individual contributions from the simple regression forecast $x_t'\delta$, the ex post inflation term $p_{t+1}-x_t'\delta$, the future terms $f_{j,t}-x_t'\delta$, and the commodity market surprise term $s_{j,t+1}-f_{j,t}-k_j$. Observations on ex post commodity forecast errors are given small weight, rarely amounting to more than 5 basis points, with the largest revision to the inference being about 10 basis points. The ex post inflation and commodity futures prices are given relatively large weight in the optimal inference about people's ex ante expectations of inflation, leading to revisions up to 53 and 45 basis points, respectively.

5. The Comparisons

The constructed inflation expectation series in the previous section using Hamilton's procedure incorporates the information available in the futures commodity markets to provide the econometrician's best inference about the people's true expectation of inflation, which may also provide a good forecast of inflation. In this section, we are interested to see whether this procedure forecasts inflation better than those derived from a univariate time series model and an interest rate model. To assess the accuracy of the constructed inflation expectation series as the forecast of inflation, we compare the Root Mean Squared Error (RMSE) statistics against the RMSE derived from the inflation forecasts generated from the time series model and the interest rate model. The RMSE is the square root of the average squared forecast error:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n}} \tag{2.51}$$

where Y_i is the observed value, $\hat{Y}_i$ is the estimated value, and n represents the number of observations in the sample. The closer the RMSE is to zero, the better are the estimates.

5.1. In-Sample Comparison

5.1.1. Univariate time series model

For comparison purposes, we still use the same CPI from the International Financial Statistics as we used in the previous section 4. To be consistent with the tri-annual data used by Hamilton, the inflation rate π_t is measured in the natural log difference of the CPI defined as $\ln(CPI_t/CPI_{t-1})$, t = I, II, III and corresponding to the value of CPI for the months of January, May and September, respectively. The data include 80 observations over the period of 1975:I to 2001:II, and are used to find the appropriate orders of an ARIMA model. After the suitable model is fitted, the inflation forecasts are calculated using the estimates.

To find the suitable time series model, the stationarity of the inflation rate series is examined first. The ADF test in Table 2.1 shows that the t-statistic is -4.03, compared with the critical value -2.89 at the 5 percent significance level, we reject the null hypothesis of a unit root in the inflation series, and conclude that the inflation series over our sample period is stationary, so the inflation rate will be modeled as an ARMA process.

The Box-Jenkins methodology is applied to select an appropriate time series model. The sample autocorrelation function (ACF) coefficients for the inflation rate (π_t) are shown in

Table 2.11, and the graphs of the ACF and partial autocorrelation function (PACF) are provided in Figure 2.3. As we can see, the ACF decays relatively slowly and the PACF shows relatively large spikes at lags 1, 2, and 3, suggesting a possible AR(3) model. The Akaike Information Criterion (AIC) and Schwartz Bayesian Criterion (SBC) are calculated and used to select the order of the ARMA(p, q) model under different specifications of $p \leq 3$ and $q \leq 3$. Both the AIC and SBC choose the model with $p = 3$ and $q = 0$. The AIC and SBC values are reported in Table 2.12.

So the ARMA(3, 0) process is fit to the inflation series. The estimated model with standard error of the estimated coefficients in parentheses is:

$$\begin{aligned} \hat{\pi}_t = \ & 1.34 + 0.25\hat{\pi}_{t-1} + 0.33\hat{\pi}_{t-2} + 0.27\hat{\pi}_{t-3} \\ & (0.563)\ (0.113) \quad (0.110) \quad (0.112) \end{aligned} \tag{2.52}$$

The diagnostic checking of the residuals from the above regression using Ljung-Box Q-Statistics confirms that the residuals are white noise. Thus the estimated model is used to construct the inflation expectation for one-period ahead using the formula in (2.19) and use which as a forecast of inflation. The RMSE value is 0.5307 over the sample period for the time series model, which is much larger than 0.2177 – the RMSE from our model when using the price level, and it is also larger than 0.4415 – the RMSE when using the inflation rate in our model. Therefore, when the econometricians are able to use information they are able to observe only after the fact, the statistically optimal inference about people's ex ante expectation of inflation is believed to be closer to the market expectation of inflation, which if used as a forecast of inflation is better than that using the inflation expectation derived from the univariate time series model as a forecast of inflation.

5.1.2. Interest rate model

The univariate time series models explain the inflation expectation according to the available past information, while another simple system constructing the inflation rate expectation is based on the classic Fisher hypothesis which concerns the connection between the nominal interest rate, real interest rate, and inflation rate. The expected rate of inflation is described as the difference between the nominal interest rate and the expected real interest rate.

Fama (1975) generated the inflation expectation treating the real interest rate as constant. Subsequent studies rejected the assumption of a constant real interest rate, and found that the expected real interest rate behaved as a random walk. To see which interest rate model is more suitable for our sample period 1975-2001, the autocorrelations of the inflation rate π_t (defined again as the natural log difference of the CPI), and the real interest rate $r_t = R_{t-1}-\pi_t$ (where R_t is the nominal Treasury bill rate obtained from the International Financial Statistics and adjusted to a monthly rate by dividing the annual rate by 12) are presented in Table 2.11.

The level of the real interest rate shows positive autocorrelations, starting with 0.60 at lag 1 and then decaying slowly. At lag 6, the ACF is still about 0.38. The stationarity of the real interest rate series is examined by the ADF test. The test statistic is -4.42. Since the critical value is -2.89 at the 5 percent level, we reject the unit root hypothesis and conclude that the real interest rate series r_t is stationary over our sample period. Both the AIC and SBC select an ARMA(3, 0) process. The values of AIC and SBC are presented in Table 2.13. The estimate of the interest rate model is:

$$\begin{aligned} \hat{r}_t = \ & -0.82 + 0.22\,\hat{r}_{t-1} + 0.33\,\hat{r}_{t-2} + 0.27\,\hat{r}_{t-3} \\ & (0.454) \quad (0.112) \quad (0.108) \quad (0.112) \end{aligned} \tag{2.53}$$

The estimate for the inflation expectation can be derived as:

$$\hat{\pi}_{t+s|t} = R_{t+s-1} - \hat{r}_{t+s|t} \tag{2.54}$$

where R_{t+s-1} is the nominal Treasury bill rate, $\hat{r}_{t+s|t}$ is the estimated real interest rate constructed using the formula in (2.19). For one-period ahead forecast (i.e., s = 1), the estimate is $\hat{\pi}_{t+1|t} = R_t - \hat{r}_{t+1|t}$. Using the estimated inflation expectation as the forecast of inflation, the in-sample RMSE for s = 1 is 0.5390, which is larger than the RMSE for the univariate time series 0.5307, larger than the RMSE 0.2177 of our model using price level and the RMSE 0.4415 when the inflation rate is used. So among the three models we used to derive the inflation expectation over 1975-2001, the interest rate model performs worst in forecasting inflation, while our model employing Hamilton's procedure performs best (no matter whether we use aggregate price in levels or in differences), and the univariate time series model performs slightly better than the interest rate model in forecasting inflation.

5.2. 'Out-of-Sample' Comparison

The method we presented using the information at the time of the forecasts, the information on the ex post price (or inflation rate) and the commodity futures data to infer the expectation of inflation using (2.44) does not exactly provide out-of-sample forecasts of inflation since the model uses the information at time t+1 to predict the inflation rate at that time. But the purpose of this comparison is to examine whether the time series model, the interest model or the model we applied will provide more accurate inflation forecast if we only use the data up to a certain point.

For the out-of-sample inflation expectation calculation, we use the observations over 1975:I to 1998:I, re-fit the corresponding model we selected previously for the univariate time series and interest rate models, then predict the one-period ahead inflation expectation. We use it as the forecast of inflation and compare it to the corresponding actual inflation rate and calculate the mean squared error; then one more observation is added, the appropriate model is fitted, another forecast of inflation and mean squared error are computed, and so on. In the end, we get ten out-of-sample forecasts of inflation, and the RMSE is calculated. For the univariate time series model, the RMSE is 0.3781, while the RMSE for the interest rate model is 0.3751. So for these two models, the interest rate model works slightly better than the univariate time series model to predict inflation, which is different from the conclusion in the in-sample comparison where the univariate time series model performs better.

The use of the commodity prices data to infer the inflation expectations and gives the RMSE value 0.1856 if we use it as the forecast of inflation employing the price level in the system of equations and following the same roll-over method as in the time series model and interest rate model. The RMSE is 0.2418 if use the inflation rate in out-of-sample prediction of inflation. We reach the same conclusion as before, which is that our model using the Hamilton's procedure provides the best estimates of inflation among three different models.[5]

6. Conclusion

In this paper, we constructed a time series of inflation expectations over the period 1975 to 2001 adopting the procedure developed by Hamilton (1992). This procedure takes into account of the information available in the future commodity markets to provide the econometrician's best inference about the people's true expectation of inflation. Then the expected inflation series is compared to the expected inflation rates derived from a simple time series model and an interest rate model as the forecast of inflation. The interest rate model gives the worst performance in in-sample forecasts while the univariate time series performs worst in out-of-sample forecasts. The Hamilton (1992) procedure performs best in both in-sample and out-of-sample comparisons over the period under investigation. We believe that the future commodity prices reflect the combined information and wisdom of millions of markets participants and provide instantaneous information about the state of economy, therefore giving signals about the future conditions of economy. The inflation expectation series we got are helpful to guide the central bank to infer the movements of future level of inflation.

Since the inflation expectation series constructed makes use not only the information available to people at the time of their forecasts, but also the information the econometricians observe after the fact, it gives the optimal inference of people's true expected rate of inflation. Such alternative, direct measure of inflation expectation series can be applied to examine different economic theories that related with expected inflation, and future works on tests of such theories may assist us to get better understanding of the economy and help the policy maker.

The expected inflation rate series constructed can be decomposed into anticipated and unanticipated components providing us the opportunity to examine whether the monetary policy change is credible. Our study represents the first attempt to analyze whether the continuous reduction of federal funds rate carried out by the Fed in early 2000s was expected when the economy went into recession. Our results indicate that the deflations are mostly anticipated by the public in the most recent economic downturn, so the Fed's monetary policies are expected by the people and hence are credible.

Notes

1. Different measures of inflation expectations are applied in these papers: the actual rate of change in the consumer price index are used by Fama (1975), Atkins (1989), Bonham (1991), Wallace and Warner (1993), and Inder and Silvapulle (1993); Carlson (1977) calculated the expected rate of inflation using the Livingston survey data of the forecasted CPI; Hess and Bicksler (1975) constructed a time series predictor of inflation based on past rates of U.S. inflation. Details on the formation of inflation expectation by previous works are presented in Section 2.2.

2. Stationarity is usually assumed for the time series applied in the regression. The ADF test is applied by considering regression $\Delta y_t = a_0 + \gamma y_{t-1} + a_2 t + \sum_i \beta_i \Delta y_{t-i+1} + \varepsilon_t$, where i=2,..,p. The lag length p in Δy_{t-i+1} is selected such that each coefficient β_i is significantly different from zero and $\{\varepsilon_t\}$ is white noise, e.g. p=4 for $\{p_t\}$.

3. Hamilton (1992) used the aggregate price levels without checking the series' stationarity.

4. The F-tests reject that the seasonal dummies have zero coefficients for both the expected and actual commodity inflations for corn and soybean at the 10 percent significance level. For oats, the coefficients for May and September are both insignificant. For wheat, expected commodity inflation shows significant seasonal effect at September, but not the actual commodity inflation. In summary, the expected commodity inflations closely track the seasonal change in the actual commodity inflations.

5. We also applied the MWSLS estimator introduced in the first chapter in the forecasting of inflation. We used the same sample over 1975:I-2001:II, and the inflation rate is defined the same as the log difference between two consecutive CPI which is tri-annual data. The inflation series is modeled without trend and with AR(3) errors. The in-sample RMSE from such model is 0.2407, which is larger than 0.2177 – the RMSE from our model when CPI is used, but smaller than 0.4415 – the RMSE from our model when inflation rate is used, 0.5307 – the RMSE from the time series model, and 0.5390 – RMSE from the interest rate model. For out-of-sample comparisons, we computed ten one-step-ahead forecasts, the RMSE from MWSLS estimator is 0.2019. The same conclusion is reached as for in-sample comparisons, that is, our model using Hamilton's procedure performs best when CPI is used, the MWSLS estimator performs slightly worse than our model. The interest rate model and the time series model perform worst, both in the in-sample and out-of-sample comparisons. In addition, we used the simple regression forecasts in equation (2.50) to forecast inflation. The calculated in-sample RMSE is 0.6582 indicating this method performed worst in all approaches, and the out-of-sample RMSE is 0.3477 which performed slightly better than the time series and the interest rate models but still worse than our model and that of using MWSLS estimator. Therefore, the use of Hamilton's procedure by incorporating additional information from commodity market beyond that in the simple forecasts improves the model's performance in forecasting inflation. In summary, we conclude that the model we applied using Hamilton's procedure produces best inflation forecast.

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Table 2.1. Unit root tests for aggregate price, the commodity spot and future prices

		Augmented Dickey-Fuller Test Statistics
Aggregate Price Index	p_t	-2.617
	Δp_t	-4.027
Commodity Spot Price	S_{corn}	-3.554
	S_{oats}	-3.331
	$S_{soybean}$	-4.018
	S_{wheat}	-3.089
Commodity Futures Price	F_{corn}	-3.440
	F_{oats}	-3.074
	$F_{soybean}$	-3.512
	F_{wheat}	-2.924

Note: The critical values are -3.51 at 1%, -2.89 at 5% and -2.58 at 10% significance level.

Table 2.2. White noise tests of the estimated error series in four-commodity system

Estimated Error Series	Commodity	Ljung-Box Q-Statistic (significance level)			
		Q(3)	Q(6)	Q(9)	Q(12)
$w_{j,t+1}$	Corn	0.465	2.486	3.972	8.480
		(0.93)	(0.87)	(0.91)	(0.75)
	Oats	0.323	6.957	13.399	14.610
		(0.96)	(0.32)	(0.15)	(0.26)
	Soybeans	4.678	9.793	12.228	13.729
		(0.20)	(0.13)	(0.20)	(0.32)
	Wheat	5.453	10.238	13.913	16.344
		(0.14)	(0.11)	(0.13)	(0.18)
$v_{j,t}$	Corn	4.906	5.872	6.916	13.623
		(0.18)	(0.44)	(0.65)	(0.33)
	Oats	5.468	7.521	9.430	12.684
		(0.14)	(0.28)	(0.40)	(0.39)
	Soybeans	7.318	10.147	12.085	13.149
		(0.06)	(0.12)	(0.21)	(0.36)
	Wheat	20.916	22.301	23.853	37.649
		(0.00)	(0.001)	(0.004)	(0.00)
$u_{j,t+1}$	-------	9.207	11.808	14.895	21.673
		(0.03)	(0.07)	(0.09)	(0.04)

Table 2.3. Seasonality regression of expected and actual commodity-price inflation

Dependent Variable	Constant Term	May Dummy	September dummy	$F_{[2,76]}$ test (p value) that seasonal dummies have zero coefficients
	Coefficient (standard error)			
Expected corn Inflation	6.377** (0.96)	-6.772** (1.37)	-4.900** (1.37)	13.084** (0.00)
Actual corn Inflation	7.143** (2.80)	-15.745** (4.00)	-8.167** (4.00)	7.745** (0.00)
Expected oats Inflation	4.369** (1.36)	-2.766 (1.95)	0.416 (1.95)	1.557 (0.22)
Actual oats inflation	0.678 (3.13)	-7.360 (4.47)	3.458 (4.47)	3.004* (0.06)
Expected soybeans inflation	3.150** (0.80)	-3.864** (1.14)	-1.488 (1.14)	5.810** (0.00)
Actual soybeans inflation	3.941 (2.92)	-8.938** (4.16)	-4.882 (4.16)	2.313* (0.10)
Expected wheat inflation	0.654 (1.11)	-0.080 (1.59)	3.433** (1.59)	3.136** (0.05)
Actual wheat inflation	-1.857 (2.68)	-0.900 (3.82)	4.493 (3.82)	1.126 (0.33)

Note: * indicates significant at 10% level, and ** indicates significant at 5% level.

Table 2.4. White noise tests of the estimated error series in three-commodity system

Estimated Error Series	Commodity	Ljung-Box Q-Statistic (significance level)			
		Q(3)	Q(6)	Q(9)	Q(12)
$w_{j,t+1}$	Corn	0.465 (0.93)	2.486 (0.87)	3.972 (0.91)	8.480 (0.75)
	Oats	0.323 (0.96)	6.957 (0.32)	13.399 (0.15)	14.610 (0.26)
	Soybeans	4.678 (0.20)	9.793 (0.13)	12.228 (0.20)	13.729 (0.32)
$v_{j,t}$	Corn	7.664 (0.05)	8.676 (0.19)	10.007 (0.35)	17.755 (0.12)
	Oats	5.113 (0.16)	6.875 (0.33)	8.203 (0.51)	10.780 (0.55)
	Soybeans	7.329 (0.06)	10.046 (0.12)	12.097 (0.21)	13.233 (0.36)
$u_{j,t+1}$	-------	9.314 (0.03)	11.575 (0.07)	15.461 (0.08)	23.684 (0.02)

Table 2.5. FIML estimation results for the three-commodity system

Likelihood values:

Unrestricted likelihood: -1109.7536

Restricted likelihood (restrictions A & B imposed): -1133.7023

Restricted likelihood (only restrictions C imposed): -1110.3976

Restricted likelihood (restrictions A, B & C imposed): -1133.8838

FIML estimates with restrictions A, B & C imposed:

$\sigma_\alpha = 0.134 \qquad \sigma_a = 0.298 \qquad q' = [0.872 \quad 2.400 \quad 2.231]$

$$\Sigma = \begin{bmatrix} 12.50 & 2.79 & 3.25 \\ 2.79 & 21.71 & 5.16 \\ 3.25 & 5.16 & 7.89 \end{bmatrix} \qquad S = \begin{bmatrix} 180.81 & 112.34 & 133.37 \\ 112.34 & 239.60 & 113.20 \\ 133.37 & 113.20 & 185.56 \end{bmatrix}$$

$$\Omega = \begin{bmatrix} 0.43 & 0.26 & 0.72 & 0.67 & 0.12 & 0.32 & 0.30 \\ 0.26 & 12.73 & 3.42 & 3.83 & 0.00 & 0.00 & 0.00 \\ 0.72 & 3.42 & 23.43 & 6.76 & 0.00 & 0.00 & 0.00 \\ 0.67 & 3.83 & 6.76 & 9.37 & 0.00 & 0.00 & 0.00 \\ 0.12 & 0.00 & 0.00 & 0.00 & 180.91 & 112.62 & 133.63 \\ 0.32 & 0.00 & 0.00 & 0.00 & 112.62 & 240.37 & 113.92 \\ 0.30 & 0.00 & 0.00 & 0.00 & 133.63 & 113.92 & 186.23 \end{bmatrix}$$

Table 2.6. Components of inferred expectations about aggregate prices

Trimester (t)	Actual Value (p_{t+1})	Expected Value ($\hat{p}^e_{t+1}$)	Contribution to $\hat{p}^e_{t+1}$ Simple Forecast ($x_t'\delta$)	Ex-post Prices ($p_{t+1} - x_t'\delta$)	Futures Term ($f_{j,t} - x_t'\delta$)	Commodity Errors ($s_{j,t+1} - f_{j,t} - k_j$)
1975:1	355.249	355.642	356.404	-0.752	-0.032	0.021
1975:2	357.907	357.608	357.459	0.291	-0.140	-0.002
1975:3	359.786	359.838	360.031	-0.160	-0.049	0.015
1976:1	361.281	361.403	362.031	-0.488	-0.128	-0.012
1976:2	363.257	363.180	363.069	0.123	0.029	-0.041
1976:3	364.832	365.030	365.262	-0.280	0.068	-0.020
1977:1	367.807	367.654	367.463	0.224	0.025	-0.058
1977:2	369.660	369.956	371.270	-1.047	-0.314	0.047
1977:3	371.382	371.449	371.590	-0.136	0.005	-0.010
1978:1	374.597	374.403	373.975	0.405	0.039	-0.015
1978:2	377.643	377.536	377.787	-0.094	-0.154	-0.004
1978:3	380.332	380.317	380.279	0.035	0.010	-0.006
1979:1	384.823	384.463	383.503	0.859	0.095	0.006
1979:2	389.060	388.932	388.483	0.375	0.060	0.014
1979:3	393.359	392.905	392.069	0.839	-0.017	0.014
1980:1	398.248	397.840	396.960	0.838	0.027	0.015
1980:2	400.988	401.237	401.650	-0.431	0.040	-0.023
1980:3	404.428	404.043	403.058	0.891	0.088	0.005
1981:1	407.635	407.616	407.442	0.126	0.041	0.008
1981:2	411.398	411.076	410.181	0.792	0.098	0.005
1981:3	412.536	412.850	413.644	-0.721	-0.065	-0.008
1982:1	414.155	414.154	414.173	-0.012	0.006	-0.013
1982:2	416.294	416.167	415.713	0.378	0.061	0.015
1982:3	416.216	416.757	417.698	-0.964	0.021	0.002
1983:1	417.577	417.564	417.286	0.190	0.072	0.018
1983:2	419.147	419.217	419.083	0.042	0.119	-0.027
1983:3	420.275	420.531	420.676	-0.261	0.104	0.011
1984:1	421.730	421.940	422.133	-0.262	0.057	0.012
1984:2	423.266	421.940	423.418	-0.099	-0.017	0.016
1984:3	423.772	423.920	424.274	-0.327	-0.028	0.000
1985:1	425.405	425.241	424.929	0.310	-0.006	0.008
1985:2	426.395	426.588	426.868	-0.308	0.004	0.024
1985:3	427.597	427.441	427.213	0.250	-0.023	0.001
1986:1	426.956	427.717	429.174	-1.443	-0.031	0.017
1986:2	428.138	427.993	427.512	0.407	0.091	-0.017
1986:3	429.019	429.042	429.080	-0.040	0.039	-0.038
1987:1	430.676	430.468	430.320	0.232	-0.069	-0.015
1987:2	432.347	432.191	432.103	0.159	-0.044	-0.027
1987:3	432.981	433.084	433.311	-0.215	0.014	-0.026
1988:1	434.523	434.458	434.254	0.176	0.029	0.000

Table 2.6. (Continued)

Trimester (t)	Actual Value (p_{t+1})	Expected Value ($\hat{p}^e_{t+1}$)	Contribution to $\hat{p}^e_{t+1}$: Simple Forecast ($x_t'\delta$)	Ex-post Prices ($p_{t+1} - x_t'\delta$)	Futures Term ($f_{j,t} - x_t'\delta$)	Commodity Errors ($s_{j,t+1} - f_{j,t} - k_j$)
1988:2	436.463	436.520	436.210	0.164	0.179	-0.034
1988:3	437.538	437.741	437.779	-0.157	0.104	0.014
1989:1	439.741	439.672	439.183	0.363	0.097	0.028
1989:2	440.709	441.103	441.582	-0.568	0.049	0.040
1989:3	442.604	442.228	441.460	0.744	0.017	0.006
1990:1	444.018	444.181	444.433	-0.270	0.016	0.002
1990:2	446.683	446.326	445.278	0.914	0.104	0.031
1990:3	448.108	448.252	448.388	-0.182	0.026	0.020
1991:1	448.852	449.214	449.829	-0.635	0.023	-0.003
1991:2	450.025	450.096	449.866	0.104	0.119	0.007
1991:3	450.679	450.747	450.793	-0.075	0.019	0.009
1992:1	451.830	451.735	451.825	0.004	-0.081	-0.012
1992:2	452.969	453.021	452.929	0.026	0.066	0.001
1992:3	453.882	453.701	453.641	0.157	-0.080	-0.016
1993:1	454.998	455.023	455.150	-0.099	-0.027	-0.001
1993:2	455.619	455.790	456.079	-0.299	0.024	-0.014
1993:3	456.372	456.391	456.238	0.087	0.048	0.017
1994:1	457.265	457.452	457.799	-0.348	-0.008	0.008
1994:2	458.538	458.494	458.475	0.041	-0.019	-0.003
1994:3	459.138	459.153	459.344	-0.134	-0.062	0.006
1995:1	460.397	460.378	460.290	0.070	0.018	0.001
1995:2	461.056	461.262	461.440	-0.250	0.076	-0.004
1995:3	461.828	461.617	461.302	0.342	0.001	-0.027
1996:1	463.249	463.079	462.846	0.263	-0.040	0.011
1996:2	464.015	463.956	464.175	-0.104	-0.123	0.008
1996:3	464.833	464.659	464.512	0.209	-0.070	0.009
1997:1	465.453	465.608	466.126	-0.437	-0.051	-0.030
1997:2	466.146	466.089	466.543	-0.259	-0.204	0.009
1997:3	466.391	466.262	466.507	-0.075	-0.166	-0.004
1998:1	467.133	467.254	467.369	-0.153	0.027	0.011
1998:2	467.619	467.813	468.125	-0.329	0.004	0.013
1998:3	468.046	468.025	467.961	0.055	0.009	0.000
1999:1	469.199	469.134	469.086	0.073	-0.034	0.009
1999:2	470.212	470.118	470.002	0.136	-0.018	-0.003
1999:3	470.682	470.616	470.610	0.047	-0.038	-0.003
2000:1	472.295	471.985	471.549	0.486	-0.032	-0.018
2000:2	473.620	473.562	473.384	0.154	0.020	0.005
2000:3	474.406	474.401	474.278	0.083	0.025	0.014
2001:1	475.875	475.706	475.454	0.274	-0.034	0.012
2001:2	476.217	476.249	476.714	-0.323	-0.112	-0.029

Note: The mean of the forecast errors for p_{t+1} is -0.005, the standard deviation is 0.219.

Table 2.7. One period ahead actual and expected inflation for period ending at indicated date

Date (t)	$3(p_{t+1} - p_t)$	$3(\hat{p}^e_{t+1} - p_t)$	Date (t)	$3(p_{t+1} - p_t)$	$3(\hat{p}^e_{t+1} - p_t)$
1975:1	6.078	7.257	1988:2	5.819	5.990
1975:2	7.972	7.076	1988:3	3.225	3.832
1975:3	5.637	5.793	1989:1	6.608	6.401
1976:1	4.484	4.852	1989:2	2.903	4.086
1976:2	5.929	5.698	1989:3	5.686	4.557
1976:3	4.724	5.320	1990:1	4.241	4.730
1977:1	8.925	8.465	1990:2	7.994	6.925
1977:2	5.559	6.447	1990:3	4.275	4.707
1977:3	5.165	5.368	1991:1	2.233	3.319
1978:1	9.644	9.064	1991:2	3.520	3.733
1978:2	9.139	8.815	1991:3	1.961	2.166
1978:3	8.068	8.023	1992:1	3.454	3.169
1979:1	13.473	12.392	1992:2	3.418	3.574
1979:2	12.710	12.327	1992:3	2.738	2.196
1979:3	12.897	11.534	1993:1	3.346	3.422
1980:1	14.667	13.442	1993:2	1.863	2.376
1980:2	8.219	8.966	1993:3	2.260	2.315
1980:3	10.320	9.164	1994:1	2.678	3.238
1981:1	9.621	9.565	1994:2	3.818	3.686
1981:2	11.290	10.322	1994:3	1.798	1.844
1981:3	3.414	4.356	1995:1	3.777	3.721
1982:1	4.856	4.853	1995:2	1.976	2.594
1982:2	6.416	6.036	1995:3	2.317	1.684
1982:3	-0.234	1.389	1996:1	4.264	3.753
1983:1	4.083	4.045	1996:2	2.298	2.121
1983:2	4.710	4.919	1996:3	2.452	1.932
1983:3	3.384	4.151	1997:1	1.861	2.324
1984:1	4.365	4.996	1997:2	2.078	1.909
1984:2	4.607	4.763	1997:3	0.735	0.347
1984:3	1.519	1.961	1998:1	2.227	2.589
1985:1	4.899	4.406	1998:2	1.457	2.039
1985:2	2.969	3.549	1998:3	1.282	1.219
1985:3	3.606	3.139	1999:1	3.459	3.265
1986:1	-1.924	0.359	1999:2	3.037	2.756
1986:2	3.545	3.111	1999:3	1.411	1.211
1986:3	2.642	2.711	2000:1	4.840	3.908
1987:1	4.972	4.346	2000:2	3.974	3.802
1987:2	5.013	4.545	2000:3	2.359	2.342
1987:3	1.903	2.211	2001:1	4.407	3.899
1988:1	4.627	4.430	2001:2	1.027	1.123

Note: The mean of the forecast errors is 0.015, and the standard deviation is 0.657.

Table 2.8. One, two and three period ahead actual and expected inflation over the period 1999-2001

Period ending	One period ahead		Two period ahead		Three period ahead	
	$3(p_t - p_{t-1})$	$3(\hat{p}^e_t - p_{t-1})$	$3(p_t - p_{t-2})$	$3(\hat{p}^e_t - p_{t-2})$	$3(p_t - p_{t-3})$	$3(\hat{p}^e_t - p_{t-3})$
1999:1	1.283	1.220	2.739	3.308	4.966	6.799
1999:2	3.458	3.265	4.741	4.208	6.197	6.272
1999:3	3.038	2.756	6.496	6.271	7.778	6.912
2000:1	1.413	1.213	4.450	3.736	7.908	7.691
2000:2	4.839	3.907	6.251	3.623	9.289	6.038
2000:3	3.974	3.801	8.812	6.561	10.225	5.646
2001:1	2.359	2.342	6.333	5.418	11.171	7.299
2001:2	4.406	3.899	6.765	5.630	10.739	8.128

Table 2.9. White noise test results of the estimated error series when using inflation rate series

Estimated Error Series	Commodity	Ljung-Box Q-Statistic (significance level)			
		Q(3)	Q(6)	Q(9)	Q(12)
$v_{j,t}$	Corn	11.163	13.431	14.521	25.009
		(0.01)	(0.04)	(0.10)	(0.01)
	Oats	5.166	7.354	8.765	12.353
		(0.16)	(0.29)	(0.46)	(0.42)
	Soybeans	14.085	18.034	19.355	24.022
		(0.003)	(0.01)	(0.02)	(0.02)
	Wheat	33.317	34.185	34.857	50.922
		(0.00)	(0.00)	(0.00)	(0.00)
$u_{j,t+1}$	-------	1.940	3.012	4.543	12.768
		(0.59)	(0.81)	(0.87)	(0.39)

Table 2.10. FIML estimation results for the three-commodity system using inflation rate

Likelihood values:

Unrestricted likelihood: -1114.4314

Restricted likelihood (restrictions A & B imposed): -1136.5614

Restricted likelihood (only restrictions C imposed): -1114.8911

Restricted likelihood (restrictions A, B & C imposed): -1136.7599

FIML estimates with restrictions A, B & C imposed:

$\sigma_\alpha = 0.271 \qquad \sigma_a = 0.127 \qquad q' = [0.252 \quad 3.415 \quad 3.285]$

$$\Sigma = \begin{bmatrix} 13.25 & 4.01 & 4.51 \\ 4.01 & 22.67 & 6.20 \\ 4.51 & 6.20 & 9.02 \end{bmatrix} \qquad S = \begin{bmatrix} 181.04 & 111.80 & 132.75 \\ 111.80 & 237.29 & 110.15 \\ 132.75 & 110.15 & 181.84 \end{bmatrix}$$

$$\Omega = \begin{bmatrix} 0.40 & 0.03 & 0.44 & 0.42 & 0.07 & 0.93 & 0.89 \\ 0.03 & 13.26 & 4.12 & 4.62 & 0.00 & 0.00 & 0.00 \\ 0.44 & 4.12 & 24.16 & 7.63 & 0.00 & 0.00 & 0.00 \\ 0.42 & 4.62 & 7.63 & 10.40 & 0.00 & 0.00 & 0.00 \\ 0.07 & 0.00 & 0.00 & 0.00 & 181.06 & 112.03 & 132.98 \\ 0.93 & 0.00 & 0.00 & 0.00 & 112.03 & 240.45 & 113.19 \\ 0.89 & 0.00 & 0.00 & 0.00 & 132.98 & 113.19 & 184.77 \end{bmatrix}$$

Table 2.11. The autocorrelation of tri-annual variables π_t, R_t, and r_t

Variable	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	ρ_8	ρ_9	ρ_{10}	ρ_{11}	ρ_{12}
π_t	0.66	0.68	0.66	0.53	0.49	0.43	0.34	0.24	0.24	0.21	0.10	0.17
R_t	0.90	0.80	0.71	0.62	0.51	0.44	0.39	0.30	0.24	0.19	0.13	0.06
r_t	0.60	0.64	0.61	0.48	0.45	0.38	0.27	0.18	0.21	0.17	0.05	0.14

Table 2.12. The AIC and SBC for the ARMA(p, q) model of inflation series

AR order	MA order	AIC	SBC	AR order	MA order	AIC	SBC
P = 0	q = 0	350.99	353.35	p = 2	q = 0	292.84	299.91
	q = 1	330.05	334.76		q = 1	291.85	301.28
	q = 2	318.16	325.23		q = 2	292.14	303.92
	q = 3	305.93	315.36		q = 3	292.17	306.31
P = 1	q = 0	307.94	312.65	p = 3	q = 0	289.21*	298.64*
	q = 1	293.82	300.89		q = 1	290.58	302.36
	q = 2	290.14	299.57		q = 2	292.01	306.15
	q = 3	290.97	302.75		q = 3	292.30	308.80

Note: * indicates the selected model.

Table 2.13. The AIC and SBC for the ARMA(p, q) model of real interest rate series

AR order	MA order	AIC	SBC	AR order	MA order	AIC	SBC
P = 0	q = 0	330.02	332.36	p = 2	q = 0	282.72	289.75
	q = 1	313.29	317.98		q = 1	281.62	291.00
	q = 2	302.97	310.00		q = 2	282.18	293.90
	q = 3	293.11	302.49		q = 3	281.70	295.76
P = 1	q = 0	296.54	301.23	p = 3	q = 0	278.73*	288.10*
	q = 1	283.63	290.67		q = 1	280.22	291.94
	q = 2	280.18	289.55		q = 2	281.86	295.92
	q = 3	280.58	292.30		q = 3	283.86	300.26

Note: * indicates the selected model.

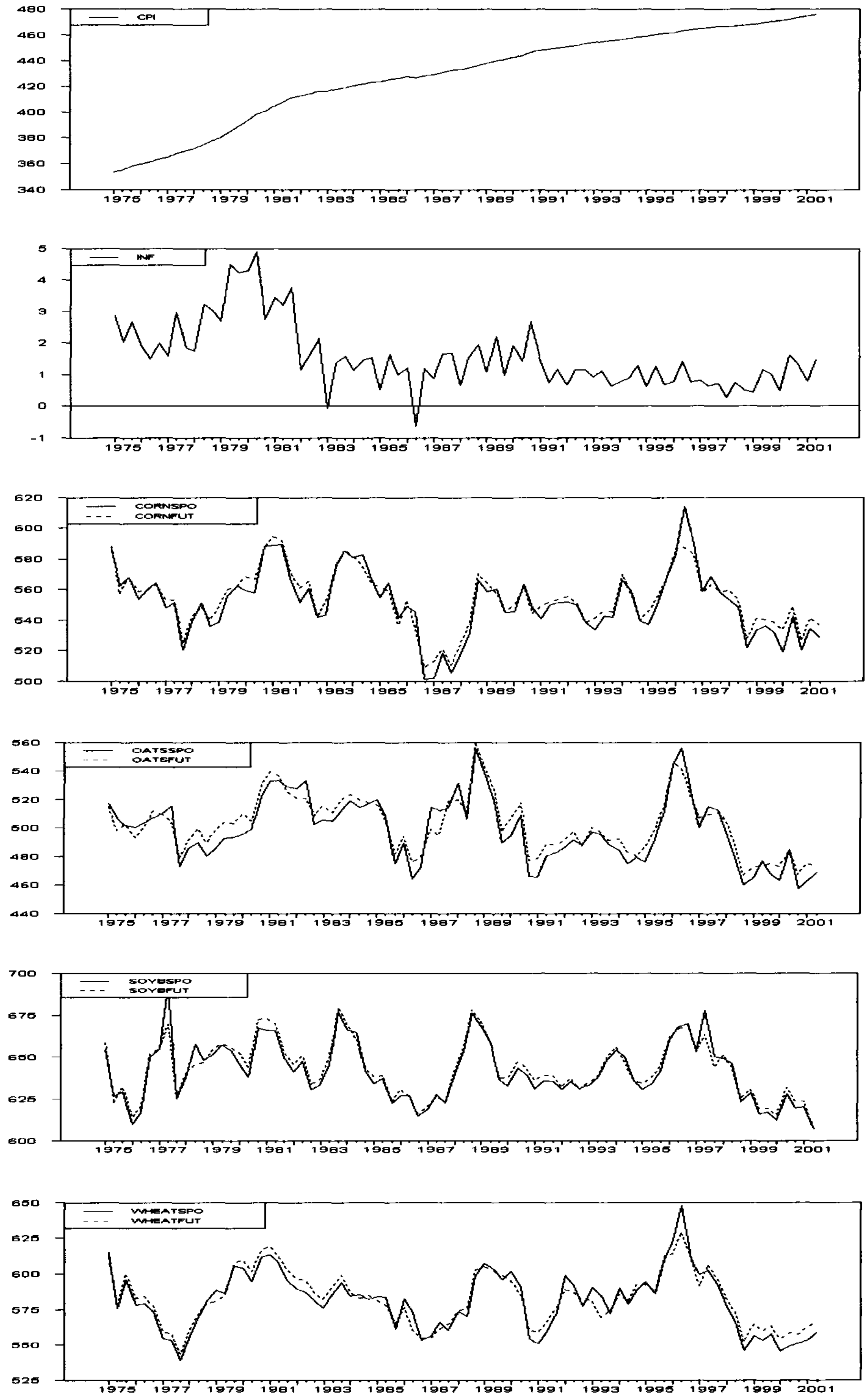

Figure 2.1. Aggregate price, commodity spot and futures price over 1975:I-2001:II

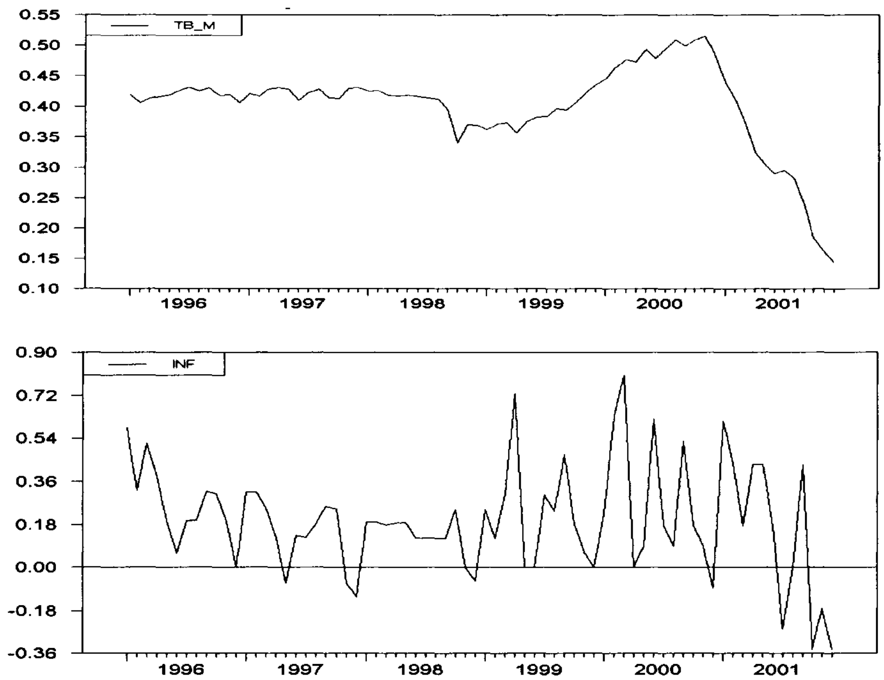

Figure 2.2. Treasury bill rate and inflation rate over 1996-2001

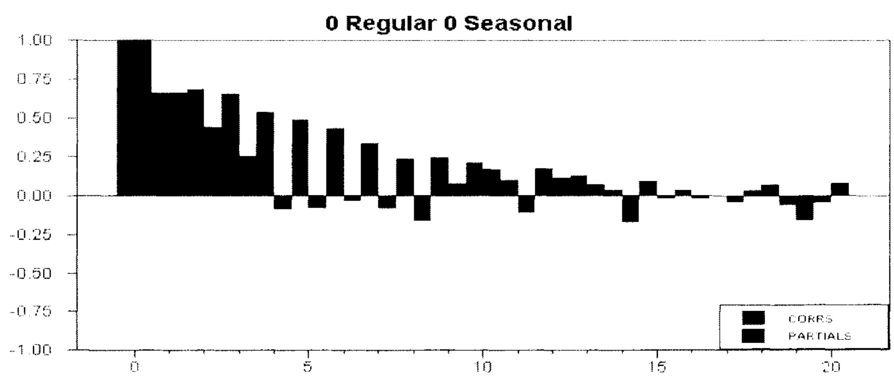

Figure 2.3. The ACF and PACF of inflation series

CHAPTER III.

AN EMPIRICAL EXAMINATION OF THE FISHER EFFECT AND THE PHILLIPS CURVE

1. Introduction

Inflation expectations play an important role in some key economic theories, such as the Fisher effect hypothesis and the Phillips curve. Although these theories are convincing on the theoretical level, there is no general consensus from empirical studies. To confirm these theories with empirical data would provide us a better understanding of the current economy, and help us predict the future conditions of the economy, thereby assisting in policy decisions and adjustments.

Since there is no direct observable data on inflation expectations, three approaches have been frequently used to measure economic agents' inflation expectations. The first approach, employed by Fisher (1930), is to derive a proxy for inflation expectations from long lags of past prices or inflation rates. The second approach, introduced by McCallum (1976), uses the actual future values of inflation as a proxy for inflation expectations. The third approach is to use expected rates of price change obtained through surveys, for instance, the Livingston survey or the Michigan survey, as a measure of inflation expectations. As pointed out by Roberts (1995, 1997), each of these three approaches has its advantages and disadvantages. The first approach is easy to implement, but is purely adaptive and only reflects information that is imbedded in the lagged inflations. The second approach does not need an explicit measure of inflation expectations, however, it introduces econometric complications due to an additional source of error[1]. The survey measure of expectations in the third approach provides a better proxy than the first two approaches because it is not purely adaptive and reflects more information than the first approach and it avoids econometric complications in using actual future inflation rates. Despite these advantages of the survey measure of inflation expectations, it has the limitation that the respondents have little incentive to provide thoughtful answers, which could result in a poor proxy for actual inflation expectations.

In our previous study, a time series of inflation expectations was obtained by applying Hamilton's (1992) procedure which incorporated information in the commodity futures market to forecast general prices. The constructed series of inflation expectations may provide a better measure of people's true expectations of price change than the purely adaptive proxy stated in the first approach and the realized future inflations used as the proxy in the second approach because the constructed series not only took into account the information imbedded in the past price experience, but also contained information reflecting instantaneous changes about the state of the economy since commodities were traded in continuous auction markets (Cody and Mills, 1991). It also avoids possible noise in survey respondents' answers for their expectations caused by using the proxy described in the third approach.

The motivation of this paper is to use this alternative measure of inflation expectations to examine two broadly debated topics in the field of economics, the Fisher effect and the Phillips curve, in which inflation expectations play a key role. The Fisher effect describes the impact of inflation expectations on the nominal interest rate, while the Phillips curve states a negative relationship between inflation and unemployment where expectations are given.

This paper is divided into four sections. The next section starts with a survey of the literature reviews on the Fisher effect, and then presents the methodology and empirical test results. Section 3 introduces two main alternative specifications of the Phillips curve augmented with inflation expectations (the expectations-augmented Phillips curve, and the New Keynesian Phillips curve), reports the estimates on the single equation models, and compares the superiority of these two specifications. Section 4 concludes this paper.

2. Fisher Effect

Irving Fisher's (1930) hypothesis about the impact of inflation expectations on nominal interest rates is one of the most studied topics in economics. Fisher (1930) claimed that in an economy with perfect foresight and a well-functioning capital market, there is a one-to-one relationship between the nominal interest rate and the rate of inflation, while the real rate of interest is unrelated to inflation being determined entirely by real factors. However, with limited information in a world of uncertainty, the exact one-to-one relationship between the nominal interest rate and the rate of inflation does not hold. Hence, there are two versions of the Fisher hypothesis in the literature: the strong form, and the weaker form. The strong form of the Fisher effect states that the nominal interest rate fully adjusts to expected inflation, while the weaker form of the Fisher effect identifies circumstances where the nominal interest rate underadjusts (or overadjusts) to inflation expectations[2].

The weaker form of the Fisher effect is more realistic in empirical studies, which can be interpreted to mean the existence of a positive relation between the nominal interest rate and expected inflation, in which case a rise in nominal interest rates does not necessarily indicate a tightening of the stance of monetary policy. Because of its importance to policy, the Fisher relationship has attracted lots of attention and been tested extensively empirically. In general, there is controversy among economists over the short-run Fisher effect (that a change in the interest rate is associated with an immediate change in the expected inflation). However, most empirical work tends to support the existence of the long-run Fisher effect (that the expected inflation rate will tend to be high when the interest rate is high for a long period of time).

In this section, we use an alternative and, we think, better measure of inflation expectations to examine the short-run Fisher effect with U.S. data. The remainder of this section consists of three subsections. We start with a survey of the literature in subsection 2.1. The methodology and the empirical test results of the short-run Fisher relationship are provided in subsections 2.2 and 2.3, respectively.

2.1. Literature Review

A problem that arises in empirical testing of the Fisher hypothesis is how to measure inflation expectations. In Fisher's (1930) original work, he assumed that the expected inflation rate is a distributed lag of current and past realized price changes, and applied a simple regression to explore the relationship between the nominal interest rate and expected inflation. Using annual data over 1890-1927 for the United States, and 1820-1924 for the United Kingdom, Fisher found that inflation expectations were not reflected instantaneously in interest rates. The lag length for the price changes was large and the lag weights dropped off slowly. So, he concluded:

> We have found evidence general and specific ... that price changes do, generallyand perceptibly affect the interest rate in the direction indicated by a priori theory. But since forethought is imperfect, the effects are smaller than the theory requires and lag behind price movements, in some periods, very greatly. When the effects of price changes upon interest rates are distributed over several years, we have found remarkably high coefficients of correlation, thus indicating that interest rates follow price changes closely in degree, though rather distantly in time. [Fisher, 1930, p451].

Other early findings in line with Fisher's initial work are that there is no relationship between the interest rate observed at a point in time and the inflation rate subsequently observed. However, there does appear to be a relationship between the current interest rate and past rates of inflation, which is interpreted as evidence in favor of the Fisherian view. For example, by adopting the basic distributed lag mechanism in the formation of expectations but using geometrically declining weights, Sargent (1969) employed a commodity price index to construct inflation expectations with U.S. annual data over the period 1902-1940. Gibson (1970) used the U.S. annual implicit NNP deflator to calculate the expected rates of price change over the period 1869-1941. Both studies supported Fisher's finding of a significant distributed lag effect in expectation formation.

The majority of the early studies used the distributed lag of price changes as a proxy for inflation expectations. However, such tests relied on the joint hypothesis that the expected rate of inflation was solely dependent upon past rates of price change, along with the Fisher effect of price expectations on market interest rates. To allow the test of the Fisher effect to be independent of the assumption that price expectations were based on past price experience, Gibson (1972) suggested using the Livingston survey data on expected rates of price change to construct inflation expectations. Using U.S. data over the period 1952-1970, he found the real rate of interest was not affected by price expectations and interest rates fully adjusted to expectations, which lent support to the Fisher effect. Pyle (1972) compared the test results of the Fisher effect using the Livingston survey data as a proxy with those using the distributed lag of price change as a proxy for inflation expectations from 1954 to 1969, and concluded that the use of the survey data was at least as powerful as the use of the distributed lag proxy in explaining nominal interest rates.

The testing of the Fisher effect took a different turn with the development of and integration between the rational expectations theory pioneered by Muth (1961) and the efficient market theory advanced by Fama (1970). In the study by Fama (1975), he argued that future price changes were reflected in the current rate of interest based on the efficient market theory, in contrast to the view that past changes in the price level were embodied in the current interest rate as suggested by Fisher (1930). Using the rate of change in Consumer Price Index (CPI) to approximate the expected rate of inflation (π_t), and the one-month Treasury bill rate as the nominal interest rate (R_t) over the period 1953-1971, Fama tested the joint hypothesis that the U.S Treasure bill market was efficient and that the expected real returns were constant through time in the following regression:

$$\pi_t = \alpha_0 + \alpha_1 R_t + \varepsilon_t \tag{3.1}$$

where α_0 represents the constant equilibrium expected real return, and the null of $\alpha_1 = -1$ states that variation in the nominal interest rate directly reflects the variation in the expected inflation rate. He showed that the estimate of α_1 was not significantly different from the hypothesized minus one, and concluded that the market used all the available information about the rate of inflation in setting the nominal rate of interest, thus supporting the Fisher effect.

Fama's findings were subsequently questioned by Carlson (1977), Hess and Bisksler (1975), and Nelson and Schwert (1977). Carlson (1977) argued the acceptance of the Fisher effect in Fama's (1975) study was because of the substantial trending in both inflation and interest rates over the sample period 1953-1971. He replicated Fama's regression by using the arithmetic average of individual forecasts of the CPI in the Livingston survey as a proxy for inflation expectations over the period 1953-1965 when the expected inflation rate was stable while the nominal interest rate had considerable variation, and found that the coefficient estimate of α_1 was close to zero and the null hypothesis $\alpha_1 = -1$ was rejected. Hess and Bisksler (1975) and Nelson and Schwert (1977) used the Box-Jenkins methodology[3] to construct optimal predictors of inflation based on the past history of inflation rates. Their regressions of the realized inflation rate on the interest rate and the time series predictor of inflation yielded a non-zero and significant coefficient for the latter term, indicating the forecasts contained information about the rate of inflation not embodied in the rate of interest, therefore rejecting the Fama's (1975) joint hypothesis.

With developments in the time series econometric literature, researchers have been forced to reconsider the validity of the previous regression tests of the Fisher effect. As pointed out by Nelson and Plosser (1982), many macroeconomic time series may be characterized as having stochastic trends, which led empirical studies on the Fisher effect to pay more attention to the development of more appropriate methods for examining the time series properties of the (expected) inflation and interest rates.

Mishkin (1992) showed that the Fisher relationship appeared only in samples where inflation and interest rates exhibited stochastic trends, which explained why a Fisher effect was widely accepted for some periods (e.g., after the Fed-Treasury Accord in 1951 until

October 1979) but not in other periods (e.g., prior to World War II or after October 1979). He also pointed out that previous studies on the Fisher effect did not make the distinction between short-run and long-run forecasting ability, hence no distinction between the short-run Fisher effect and the long-run Fisher effect. Mishkin (1992) suggested, the correct procedure to test the long-run Fisher effect, when inflation and interest rates had unit roots, was to test for cointegration between π_t^m *and* i_t^m in the following equation:

$$E_t[\pi_t^m] = \alpha_m + \beta_m i_t^m + u_t^m \tag{3.2}$$

where π_t^m is the m-period future inflation rate from time t to t+m, and i_t^m is the m-period interest rate known at time t.[4] The test for the short-run Fisher effect when both series had unit roots was to test for a significant positive coefficient β_m in the following regression:

$$E_t[\pi_t^m] - E_{t-1}[\pi_{t-1}^m] = \alpha_m + \beta_m[i_t^m - i_{t-1}^m] + v_t^m \tag{3.3}$$

which could be modified under rational expectations as:

$$\Delta\pi_t^m = \alpha_m + \beta_m \Delta i_t^m + \eta_t^m \tag{3.4}$$

where $\eta_t^m = v_t^m + \varepsilon_t^m - \varepsilon_{t-1}^m$; and ε_t^m is defined as $\varepsilon_t^m = \pi_t^m - E_t[\pi_t^m]$, which is the forecast error of inflation that is orthogonal to any information known at time t. Using monthly data on inflation and Treasury bill rates from 1953 to 1990, Mishkin (1992) found unit roots in both the inflation and interest rate series, and evidence of the long-run Fisher effect using the Engle and Granger (1987) cointegration procedure. However, he did not find evidence of a short-run relationship. In contrast, Wallace and Warner (1993) found a short-run Fisher effect, and a long-run Fisher effect by applying the Johansen cointegration test (Johansen, 1988; Johansen & Juselius, 1990), with U.S. quarterly data over the period 1948-1990. Evans and Lewis (1995) found that the Johansen cointegrating estimates differed from the hypothesized values and hence invalidated the long-run Fisher effect over the period 1947-1987. They argued, however, that such a conclusion was deceptive because rational expectations of infrequent shifts in the inflation process induced small sample bias that persisted even in fairly long samples. By characterizing possible shifts in inflation as a Markov switching model and using the forecasts of inflation expectations from such a model to test the long-run Fisher effect, they were unable to reject that the nominal interest rate reflected the expected inflation rate one-for-one in the long run.

A generalized form of the Fisher equation taking into consideration the tax effect on the nominal return was studied by Crowder and Hoffman (1996). They stated that the "after-tax" nominal interest rate was positively related to the real rate and expected inflation under rational expectations as in Fisher's original theory:

$$(1-\tau_t)i_t = \alpha + \beta\pi_t + \varepsilon_t \tag{3.5}$$

where τ_t is the tax rate on nominal returns, and the actual inflation rate π_t is used as a proxy for inflation expectations. They applied the Johansen procedure to test cointegration between the rate of interest and the rate of inflation using U.S. quarterly data over 1952-1991. They observed that a one percent increase in inflation yielded a 1.34 percent increase in the nominal interest rate before adjusting for the tax effect; after the tax-effect adjustment, the Fisher effect was insignificantly different from unity, as implied by the theory.

In 1997, Weidmann extended the literature on the long-run Fisher effect in a new direction. He proposed a threshold cointegration (TC) model, which is a nonlinear model and can account for the fact that inflation and interest rates seldom occur outside some narrow band in some industrialized countries.[5] Weidmann (1997) applied the TC model to test the Fisher relationship with German data over the period 1967-1996, and found evidence supporting the Fisher effect that nominal interest rate vary one-for-one with inflation in the long run. A more recent paper by Bajo-Rubio, Diaz-Roldan and Esteve (2004) made use of the TC model in the analysis of the role of nonlinearity in the Fisher relationship for Spain over the period 1963-2002, and found evidence in favor of a two-regime TC model which characterized the nonlinear adjustment of the nominal interest rate toward a long-run equilibrium.

Summing up, empirical studies of the Fisher effect hypothesis have tried various approaches to derive a proxy for inflation expectations over the years, from using a distributed lag on past price change as a proxy in early studies to using directly observed inflation expectations from surveys in later work. Recent studies focus more on methodological advances involving the examination of the time series properties of expected inflation and interest rates, along with the nonlinearity in the Fisher relationship. Overall, empirical studies on the Fisher relationship have provided mixed, inconsistent results on the short-run Fisher effect, but tend to support the existence of the long-run Fisher effect. Our purpose in this study is to focus on using an alternative measure of inflation expectations, which is considered to be able to track more closely any expected change in inflation, in the examination of the short-run Fisher effect with U.S. data. In the next subsection, we introduce the statistical tests for the presence of a unit root, and the empirical procedure that will be used to examine the short-run Fisher effect.

2.2. Methodology

2.2.1. Unit root tests

As pointed out by Granger and Newbold (1974), and Phillips (1986), a regression may be spurious in the presence of unit root variables. That is, the usual estimation and inference procedures will tend to suggest a statistically significant relationship between variables that are, in fact, generated independently of one another. Therefore, in the study of the Fisher effect, we need to examine whether the expected inflation rate and the interest rate are unit root processes. Some researchers, including Mishkin (1992), Wallace and Warner (1993), and Evans and Lewis (1995) have shown that the null hypothesis that the inflation and interest rates contain a unit root cannot be rejected for U.S. data. Two formal tests, the Augmented Dickey-Fuller (ADF) test and the Phillips-Perron test, are used to check for the presence of a unit root in this study.

Dickey and Fuller (1979, 1981) considered three different regression equations in the first-order autoregressive process to test for the presence of a unit root. The Dickey-Fuller (DF) test was later extended to test a unit root in higher-order equations (i.e., p-th order autoregressive processes), and is referred to as the ADF test:

$$\Delta y_t = \rho y_{t-1} + \sum_{i=2}^{p} \phi_i \Delta y_{t-i+1} + \varepsilon_t \tag{3.6}$$

$$\Delta y_t = \alpha + \rho y_{t-1} + \sum_{i=2}^{p} \phi_i \Delta y_{t-i+1} + \varepsilon_t \tag{3.7}$$

$$\Delta y_t = \alpha + \rho y_{t-1} + \beta t + \sum_{i=2}^{p} \phi_i \Delta y_{t-i+1} + \varepsilon_t \tag{3.8}$$

The unit root null hypothesis is, in each case, H_0: $\rho = 0$. The autoregressive terms, Δy_{t-1}, …, Δy_{t-p+1}, account for serial correlation in the y_t process. Aside from the autoregressive terms, regression equation (3.6) represents a random walk model under the null and y_t is a zero-mean stationary process under the alternative. Equation (3.7) adds an intercept which is assumed to be zero under H_0. Thus, under the null y_t is a random walk but it can have a non-zero mean under the alternative. Equation (3.8) includes a linear time trend. Under the null, β is assumed to be zero but α is unrestricted so that y_t is a random walk with drift. Under the alternative, y_t is trend stationary. Thus, the proper choice of model (3.6), (3.7), or (3.8) depends on the kind of behavior one wants to allow under the null and alternative hypotheses. The standard DF and ADF tests are based on the t-statistic for the OLS estimator of ρ. In all of the above regression equations, if we cannot reject the null hypothesis H_0: $\rho = 0$, then the conclusion is that the time series $\{y_t\}$ contains a unit root, i.e., $\{y_t\}$ is a nonstationary process. Dickey and Fuller (1981) also provided F-statistics, called ϕ_1, ϕ_2, and ϕ_3, to test joint hypotheses on the coefficients. Using equation (3.7), the null hypothesis $\rho = \alpha = 0$ is tested using the ϕ_1 statistic. The joint hypothesis $\rho = \alpha = \beta = 0$ in equation (3.8) is tested using the ϕ_2 statistic, and the null $\rho = \beta = 0$ is tested using the ϕ_3 statistic. The ADF test assumes that the errors are statistically independent and have a constant variance. To relax such assumptions and allow for autocorrelation and heteroskedasticity in the error term of the Dickey-Fuller regressions, Phillips and Perron (1988) suggested a nonparametric correction to the DF test, referred to as the Phillips-Perron test.

2.2.2. Tests for the Fisher relationship

The Fisher effect can be summarized in mathematical terms as:

$$i_t = E_{t-1}[r_t] + E_{t-1}[\pi_t] \tag{3.9}$$

where i_t refers to the nominal interest rate at time t; $E_{t-1}[r_t]$ and $E_{t-1}[\pi_t]$ are respectively the real rate of interest expected, and the inflation rate expected at time t-1 for the period t.

Based on the assumption that the real interest rate does not change much, the expected real rate can be expressed as the difference between a constant α_0, and a stationary disturbance ε_t:

$$E_{t-1}[r_t] = \alpha_0 - \varepsilon_t \tag{3.10}$$

Substitute it into equation (3.9) and rearrange:

$$E_{t-1}[\pi_t] = \alpha_1 + i_t + \varepsilon_t \tag{3.11}$$

where $\alpha_1 = -\alpha_0$. If there is a Fisher effect such that the impact of expected inflation on the nominal interest rate exists, the best linear forecast of expected inflation can be obtained by regressing the expected rate of inflation on the nominal interest rate:

$$\pi_t^e = \alpha + \beta\, i_t + \varepsilon_t \tag{3.12}$$

where $\pi_t^e = E_{t-1}[\pi_t]$, and ε_t represents the error term. A significant β coefficient indicates the existence of the weaker form of the Fisher effect, while a unity means the existence of the strong form of the Fisher effect.

The regression in equation (3.12) may be spurious if any of the variables contain a unit root. When both the expected inflation and the interest rate have a unit root, the short-run Fisher effect can be tested using the following regression:

$$\Delta\pi_t^e = \alpha + \beta\Delta i_t + \eta_t \tag{3.13}$$

where $\Delta\pi_t^e = E_{t-1}[\pi_t] - E_{t-2}[\pi_{t-1}]$, $\Delta i_t = i_t - i_{t-1}$, and η_t represents the disturbance term. The finding of a positive significant coefficient β in the above equation would suggest evidence in favor of the short-run Fisher effect that a change in the interest rate is associated with an immediate change in the expected inflation rate.

2.3. Empirical Results

The expected rate of inflation, π_t^e, is obtained from a procedure proposed by Hamilton (1992) taking into account tri-annual commodity futures market data from mid-1975 to the end of 2001. The nominal interest rate series, i_t, is the three-month Treasury bill rate taken from the databank of the Federal Reserve Bank at St. Louis. To match the tri-annual frequency of the expected rate of inflation, we choose the January, May and September monthly interest rates, which we refer to as I, II and III, respectively. The WINRATS program is used in this study.

Plots of the expected rate of inflation and the interest rate series appear in Figure 3.1. It seems there is little evidence of explosive behavior or a time trend; however, both series appear to be more volatile around the period 1979-1982[6]. It is widely accepted that the monetary regime changed at 1979 and early 1980s. Results in previous studies suggested that the relationship of nominal interest rates and inflation shifted with the monetary regime change (Clarida & Friedman, 1984; Huizinga & Mishkin, 1986; Roley, 1986; Mishkin, 1992).

The stationarity of the expected rate of inflation and the interest rate series are examined by the ADF and the Phillips-Perron tests. Panel A of Table 3.1 reports the results of the ADF tests. The lag length in the ADF test is selected by starting with higher-order lags, for instance, n, then delete the insignificant lag length using the usual t-test and re-run the regression with n-1 lags; repeat this process until the last lag is significantly different from zero. The number of lags used is three for the expected rate of inflation, and one for the interest rate series. The ADF test statistics for each variable and for the different specifications described in equations (3.6) to (3.8) are all insignificantly different from zero at the 5 percent level regardless of the presence of a constant or a trend. So we are unable to reject the null hypothesis of a unit root in any case and conclude that the expected inflation rate and the interest rate are unit root processes.

Panel B of Table 3.1 shows the Phillips-Perron test statistics with different lag lengths ranging from one to three. The results confirm the conclusions from the ADF test that the expected rate of inflation and the interest rate series possess a unit root in their autoregressive representation[7].

As we mentioned earlier, there are noted monetary policy changes at 1979 and early 1980s. Unit root tests are biased towards accepting the null of a unit root in the presence of a structural break (Enders, 1995), so we need to be cautious about the conclusions that the expected rate of inflation and the interest rate series are nonstationary over 1975:II-2001:III. To avoid possible structural breaks, we restrict our sample to 1982-2001, and apply the ADF and the Phillips-Perron tests to examine whether the two time series are unit root processes over this subsample period.

Over the sample period 1982-2001, the number of lags used for the ADF test is two for the expected rate of inflation, and one for the interest rate series. The ADF test statistics for each variable and for the different specifications are all significantly different from zero at the 5 percent level regardless of the presence of a constant or a trend. So we reject the null of a unit root and conclude that the expected rate of inflation and the interest rate series are stationary processes. The Phillips-Perron test statistics tell the same story that these two series do not contain a unit root at the 10 percent significance level. Therefore, to make the sample more homogeneous and avoid possible structural breaks, our analysis below are conducted over 1982:I-2001:III under the assumption that the expected rate of inflation and the interest rate series are stationary.

The short-run Fisher effect says that a change in the interest rate is associated with an immediate change in the expected inflation. Since both the expected rate of inflation and the interest rate series are stationary, the test of the short-run Fisher effect involves testing for a significant correlation of the level of interest rates and the expected inflation, i.e., testing for the significance of β in equation (3.12) in the weaker form of the Fisher effect, and one in the strong form of the Fisher effect. The parameter estimates with the standard errors given in parentheses are:

$$\begin{array}{rl} \pi_t^e = & 1.144 + 0.360 i_t \\ & (0.473) \quad (0.074) \end{array} \tag{3.14}$$

The point estimate of β is 0.36, which is significantly different from zero at the 5 percent level. However, the Ljung-Box Q-statistics indicate the presence of serial correlation in the regression errors[8], so the usual OLS test statistics are not valid.

There are two approaches to deal with the serial correlation problem. The first approach is to re-estimate equation (3.12) by the feasible generalized least squares (FGLS) procedure. An AR(3) model, as suggested by the Box-Jenkins methodology, seems adequate to represent the serial correlation:

$$\begin{array}{rl} \hat{\varepsilon}_t = & -0.024\hat{\varepsilon}_{t-1} + 0.170\hat{\varepsilon}_{t-2} + 0.391\hat{\varepsilon}_{t-3} \\ & (0.121) \qquad (0.121) \qquad (0.122) \end{array} \tag{3.15}$$

where $\hat{\varepsilon}_{t-1}$, $\hat{\varepsilon}_{t-2}$ and $\hat{\varepsilon}_{t-3}$ are the lagged terms of the estimated residuals. Let the coefficient estimates of the AR(3) model above be $\hat{\rho}_1$= -0.024, $\hat{\rho}_2$= 0.170, and $\hat{\rho}_3$= 0.391, then transform the variables in equation (3.12) as:

$$\tilde{\pi}_t^e = \pi_t^e - \hat{\rho}_1 \pi_{t-1}^e - \hat{\rho}_2 \pi_{t-2}^e - \hat{\rho}_3 \pi_{t-3}^e \tag{3.16}$$

$$\tilde{i}_t = i_t - \hat{\rho}_1 i_{t-1} - \hat{\rho}_2 i_{t-2} - \hat{\rho}_3 i_{t-3} \tag{3.17}$$

Next, estimate the equation by the OLS using the transformed variables:

$$\begin{aligned} \tilde{\pi}_t^e &= 0.337 + 0.407\, \tilde{i}_t \\ &\quad (0.322) \quad (0.114) \end{aligned} \tag{3.18}$$

The estimated coefficient for the interest rate term is about 0.41, which is significantly different from zero at the 5 percent level. The diagnostic checking by the Ljung-Box Q-statistics indicates that there is no serial correlation in the estimated regression errors in the above regression[9]. The second approach attempts to construct an autocorrelation-robust estimated standard error. The parameter estimates with the estimated robust errors given in parentheses are:

$$\begin{aligned} \pi_t^e &= 1.144 + 0.360 i_t \\ &\quad (0.535) \quad (0.083) \end{aligned} \tag{3.19}$$

The estimated coefficient for the interest rate term is significantly different from zero at the 5 percent level, which is consistent with the result when the FGLS approach is used. Therefore, the preceding empirical results indicate that the three-month Treasury bill rates contain a highly significant amount of predictive power for expected inflation, which in turn suggests the existence of the short-run Fisher effect over the time period 1982-2001.

The conclusion regarding the validity of the short-run Fisher effect is based on the estimation using the constructed expected rate of inflation incorporating information in the commodity futures market to measure inflation expectations. In our previous study, another proxy for inflation expectations was derived from an ARMA(3,0) time series model. Using this proxy in the examination of the short-run Fisher effect, the parameter estimates from the FGLS are[10]:

$$\tilde{\pi}_t^e = \underset{(0.065)}{0.810} + \underset{(0.082)}{0.090}\,\tilde{i}_t \tag{3.20}$$

The point estimate of β is not significantly different from zero at the 5 percent level, suggesting that there is no evidence for the presence of a short-run Fisher effect, which is in contrast to the conclusion when the constructed expected rate of inflation is used as the measure of inflation expectations. However, if we construct an autocorrelation-robust estimated standard error, the parameter estimate of β is significantly different from zero at the 5 percent level in the estimated equation:

$$\pi_t^e = \underset{(0.878)}{0.996} + \underset{(0.142)}{0.424}\,i_t \tag{3.21}$$

This provides supporting evidence for the short-run Fisher effect, which contradicts the conclusion when we use the FGLS approach to correct the serial correlation. So it seems that the choices of how to correct the serial correlation affects our conclusion on the test of the short-run Fisher effect when we use the measure of inflation expectations derived from the time series model, and we are not clear about the sources that cause these different conclusions.

Since the proxy obtained from the time series model is purely adaptive and reflects only information imbedded in the past history of inflation rates, while the constructed inflation expectations using Hamilton's procedure take into account not only the information available at the time of people's forecast but also the information inferred by econometricians after the fact which provides a better measure of people's true expectations, therefore we believe the conclusion of the existence of the short-run Fisher effect when using the constructed inflation expectations using Hamilton's procedure is more convincing.

Summing up, we find a positive relation between expected inflation and the nominal interest rate, which provides supporting evidence for the existence of the short-run Fisher effect when the expected rate of inflation that is believed to be a more accurate approximation to people's true expectations of inflation is employed in the examination of the short-run Fisher effect.

3. Phillips Curve

Mankiw (1998) summarized ten essential principles in economics. One of the three principles allocated to macroeconomics is: "Society faces a short-run tradeoff between inflation and unemployment" which is a statement about the effects of monetary policy and claims that changes in monetary policy push these two variables in opposite directions. Such a negative relationship was hypothesized originally by Phillips (1958) and Samuelson and Solow (1960). Empirically, it is described by the conventional Phillips curve: the higher the rate of unemployment, the lower the rate of inflation. The conventional Phillips curve held pretty well throughout the 1960s in the United States.

At the height of the conventional Phillips curve's popularity as a guide to policy, Phelps (1967) and Friedman (1968) independently challenged its theoretical foundations. They argued that well-informed, rational employers and workers would pay attention only to the real wage rate rather than the nominal wage rate, so the effects of expected inflation on wage bargaining and price settings should be taken into account. An expectations-augmented

Phillips Curve was proposed with a central role placed for expectations in the inflation process. The coexistence of high unemployment and high inflation in the 1970s supported the Phelps and Friedman arguments, and the failure of the conventional Phillips curve was attributed to the failure to consider the role of expectations.

Later, building on the theoretical modeling work of Taylor (1980), Rotemberg (1982), and Calvo (1983), the New Keynesian Phillips curve was specified. The New Keynesian Phillips curve suggests that prices are sticky and the expected future inflation at the current time determines current inflation. This is contrary to the expectations-augmented Phillips curve, where the expected future inflation in the previous period determines the current inflation.

The expectations-augmented Phillips curve and the New Keynesian Phillips curve are two main alternative specifications of the Phillips curve augmented with expectations. In general, the two Phillips curves are specified in terms of unemployment, but alternative measures of economic activity (e.g. the output gap, or capacity utilization) can be used. Then the Phillips curve is interpreted broadly as a relation between aggregate real economic activity and expected inflation.

Paloviita (2002) assessed empirically the two main alternative specifications of the Phillips curve, specified in terms of the output gap for the Euro area over the period 1981-2000. He used direct measures of inflation expectations obtained from the OECD inflation forecasts, and found that the New Keynesian Phillips curve fit the data slightly better than the expectations-augmented Phillips curve.

Following the study of Paloviita (2002), the aim of this paper is to estimate and compare the two main alternative specifications of the Phillips curve specified in terms of the unemployment gap for the United States. Instead of using survey proxies for inflation expectation as in Paloviita's paper, we measure inflation expectations by a constructed time series of the expected rate of inflation. As we have shown before, the time series of the expected rate of inflation constructed from Hamilton's (1992) procedure seems to provide a good empirical proxy of people's inflation expectations. For each of the two main alternative specifications of the Phillips curve, the Phillips curve equation is first estimated using the generalized method of moments estimator. Then, the empirical superiority of these two specifications is examined by encompassing and non-nested tests. We hope to cast some light on the modeling of inflation dynamics by addressing which specification fits U.S. data better, which, in turn, helps provide guidance for forecasting and policy analysis.

The remainder of this paper proceeds as follows. A survey of the literature on the two main alternative specifications of the Phillips curve is presented in section 3.1. Section 3.2 provides an overview of the methodologies used to estimate the Phillips curve equation and compare the two main specifications. Then, in section 3.3, the data are introduced and the empirical results are reported.

3.1. Literature Review

In 1958, Phillips published a study which represents a milestone in the development of macroeconomics. He showed that there was a consistent negative relationship between the rate of wage inflation and the rate of unemployment in the United Kingdom over the period 1861-1957. The only important exception was during the period of volatile inflation between the two World Wars.

Samuelson and Solow (1960) estimated the conventional Phillips curve with U.S. data from 1935 to 1959. They found similar results to those of Phillips (1958), confirming the existence of an inverse relationship between wage inflation and unemployment for the U.S. economy. The conventional Phillips curve performed well in the United States during the 1960s, which encouraged many economists to "treat it as a sort of menu of policy trade-offs" (Hoover, 2003). However, the conventional Phillips curve failed to explain the coexistence of high unemployment and high inflation, i.e., the stagflation, in the 1970s.

The breakdown of the conventional Phillips curve as an empirical relationship was preceded by theoretical arguments predicting its failure. In the late 1960s, Phelps (1967) and Friedman[11] (1968) independently challenged the conventional Phillips curve's theoretical underpinnings, and proposed an expectations-augmented Phillips curve. In their version, suppliers of labor at the beginning of an inflationary period underestimate the price level that would prevail over the period of the work contract, and offer a greater supply of labor according to the overestimated real wage at the prevailing (nominal) wage than they would if expectations were correct, which results in employment greater than the equilibrium level. The low unemployment signals a tight labor market, wages tend to rise or rise more quickly than expected as wages are being bid upwards when employers attempt to fill out the rank of their required labor force. Since the wages are the major cost of production, rising wages lead to the increase of prices. In short, a fall in unemployment leads to a rise in inflation in the short-run where expectations are given. By assuming that current inflation depends negatively on the amount of slack in the labor market, the expectations-augmented Phillips curve simply translates labor market slack, unemployment, into inflation:

$$\pi_t = \pi_t^e + \lambda\,(u_t - u_t^*) \qquad (3.22)$$

where u_t is the observed unemployment rate; u_t^* is the non-accelerating-inflation rate of unemployment (NAIRU[12]); and the difference between u_t and u_t^* is the unemployment gap, which measures the slack in the labor market. A negative (positive) unemployment gap indicates that the economy is operating above (below) potential output, and represents the excess demand for (supply of) labor and/or output in economy. The variable π_t^e is the expected inflation rate, in the expectations-augmented specification of the Phillips curve, it stands for $E_{t-1}[\pi_t]$ which is the expected rate of inflation formed at the end of period t-1 for period t. The parameter λ is assumed to be less than zero, based on the assumption that current inflation depends negatively on the amount of slack in the labor market. In empirical estimation, equation (3.22) is modified slightly as:

$$\pi_t = \beta E_{t-1}[\pi_t] + \lambda (u_t - u_t^*) \tag{3.23}$$

where β is expected to be one. Other measures instead of unemployment gap have been used in empirical studies, for example, the capacity utilization rate and the output gap[13].

The expectations-augmented Phillips curve specification has been assessed and employed in many empirical studies. Tootell (1994) employed an expectations-augmented Phillips curve to test the stability of the U.S. NAIRU. Using quarterly data from 1973 to 1993 and weighted average of lagged inflation rates as a proxy for inflation expectations, he found little support for a significant change in the NAIRU. The estimated coefficient for the unemployment gap was negative and significant, providing additional evidence for the Phillips curve relation. Using U.S. state-level data covering the period 1964-1993, Payne (1995) tested the expectations-augmented Phillips curve with both adaptive and rational expectations estimates[14] of the expected rate of inflation, and concluded that the expectations-augmented Phillips curve based on adaptive expectations was preferred. Lown and Rich (1997) pointed out that inflation had not accelerated since the 1990-1991 recession, and remained stable from late 1993 to early 1995[15]. They also noted the unusual slowdown in compensation growth[16] during the period 1992-1994, which led them to investigate whether the slow compensation growth could account for the low level of inflation since labor costs are an important factor in determining prices. By including compensation growth in their model, Lown and Rich showed that the modified Phillips curve was able to track inflation more accurately. Thus, the slow compensation growth resulted in the coexistence of low unemployment and low inflation over the post-1991 period, and they concluded that the Phillips curve was inherently stable. Eliasson (2001) assessed the evidence of instability and nonlinearity in the parameters of the expectations-augmented Phillips curve for the United States. She could not reject the linearity and the stability of the Phillips curve using quarterly data from 1978 to 1997 when the Michigan survey measure of expected inflation was used as a proxy for inflation expectations.

Advances in the theoretical modeling of inflation dynamics formed around the microfoundations of sticky price models led to the specification of the New Keynesian Phillips curve. One category of the sticky price models in the New Keynesian literature is the time-dependent model[17] built upon the Taylor's (1979, 1980) work on overlapping contracts and Rotemberg's (1982) model of quadratic costs of price adjustment. The most elegant formulation is based on Calvo's (1983) model of random price adjustment, which is introduced briefly below.

In Calvo's model, firms follow time-contingent price adjustment rules in which there is no deterministic schedule for price adjustment but the time for adjustment arrives randomly from a Poisson process. Every period, a fraction of firms adjust prices, and each firm has the same probability (1-θ) of being one of the adjusting firms regardless of how long it has been since its last price adjustment. Correspondingly, each firm has the probability θ of keeping the price fixed. Thus, the degree of nominal rigidity in the economy increases as θ rises. Calvo's model states that firms set their prices for fixed periods of time, so that prices are sticky and inflation depends entirely on current and expected future economic conditions:

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa(u_t - u_t^*) \tag{3.24}$$

where $E_t[\pi_{t+1}]$ refers to the expected rate of inflation π_{t+1}^e for period t+1 at time t; the parameter β is the discount factor; and the coefficient κ represents the sensitivity of inflation to variation of labor slack[18].

Empirical studies of the New Keynesian model have not provided consistent evidence on the Phillips curve relation for the United States. Roberts (1995) presented estimates of the New Keynesian Phillips curve over the period 1949-1990, and concluded that the New Keynesian Phillips curve was stable. In addition, he suggested that the actual future inflation was a worse proxy for inflation expectations than were the surveys, based on the evidence that the coefficient estimate for the excess demand variable was significant and had the expected sign when the Michigan survey and the Livingston survey of price expectations were used as proxies, while the coefficient estimate was not significant due to large estimated standard error when the actual future inflation was used as a proxy. In a later study, Roberts (1997) analyzed inflation dynamics over the period 1961-1995, and found favorable evidence for the New Keynesian Phillips curve when the Michigan survey estimate of inflation expectations was employed in the estimation. By contrast, Fuhrer and Moore (1995) argued that the standard New Keynesian model with sticky prices and rational expectations did not fit U.S. post-war data from 1965 to 1993.

In 1999, Gali and Gertler introduced a new hybrid Phillips curve allowing a fraction of firms to use a backward-looking rule to set prices. The New Keynesian Phillips curve with only forward-looking elements is nested within the hybrid Phillips curve as a special case:

$$\pi_t = \lambda s_t + \gamma_f E_t[\pi_{t+1}] + \gamma_b \pi_{t-1} \tag{3.25}$$

where s_t is a measure of real marginal cost defined as the real unit labor costs in the non-farm business sector; π_{t-1} is the lagged term of inflation that is designed to capture the inflation persistence. The hybrid Phillips curve model reduces to the New Keynesian Phillips curve when the coefficient $\gamma_b = 0$. The GMM estimates of the model suggested that the New Keynesian Phillips curve provided a reasonably good description of quarterly U.S. inflation dynamics over the period 1960-1997, so that forward-looking behavior was more important than backward-looking behavior. However, using U.S. data over the same period, Rudd and Whelan (2002) estimated a hybrid model by an alternative GMM procedure and found a very limited role for forward-looking behavior. Linde (2002) made use of the FIML method and found that backward-looking behavior was more important, though forward-looking behavior was highly significant. By contrast, Kurmann (2003) extended a Maximum Likelihood approach to estimate the hybrid Phillips curve, and obtained similar results to those reported by Gali and Gertler (1999). Furthermore, Gali, Gertler and Lopez-Salido (2003) re-examined the hybrid Phillips curve using a variety of econometric procedures, and claimed that the conclusions of Gali and Gertler (1999) and others regarding the importance role of forward-looking behavior appeared to be robust.

Such studies, by examining whether forward-looking behavior or backward-looking behavior is more appropriate in describing inflation dynamics, are helpful in policy analysis to determine whether inflation can be controlled through the expectation channel. In our analysis, we emphasize on the role of expectations in inflation dynamics by assuming forward-looking behavior of economic agents, and examine whether the previously expected current inflation or the current expected future inflation (i.e., the expectations-augmented Phillips curve or the New Keynesian Phillips curve) dominates inflation dynamics. Our study might provide a guide to monetary policy decisions since under the expectations-augmented Phillips curve, monetary policy changes inflation indirectly through excess demand, while under the New Keynesian Phillips curve, a transition to a new policy regime affects inflation immediately. To our knowledge, direct comparison of the expectations-augmented Phillips curve and the New Keynesian Phillips curve specified in terms of unemployment gap has not been presented for the United States in the existing literature.

In the next section, we describe our estimation procedures. In the subsequent one, we first present estimates of the expectations-augmented Phillips curve and the New Keynesian Phillips curve, then the comparison tests of the two main alternative specifications of the Phillips curve are provided to examine which specification is better in describing U.S. inflation dynamics.

3.2. Testing Strategy

3.2.1. Single equation estimation

Following Paloviita (2002), the econometric procedure we use to examine each of the two main alternative specifications of the Phillips curve is the Generalized Method of Moments (GMM). The GMM approach is often employed in empirical investigation of the Phillips curve relation, for instance, Gali and Gertler (1999), Roberts (2001), and Gali, Gertler and Lopez-Salido (2001, 2003).

Consider the following model:

$$y = X\xi + \varepsilon \tag{3.26}$$

where y is a T×1 vector of observations; X is a T×K matrix of observations; ξ is a K×1 vector of parameters to be estimated; and ε is a T×1 vector of disturbances. In our analysis, each specification of the Phillips curve can be written in the form of equation (3.26), where y is the inflation rate π_t; X is a T×2 matrix, where two columns are, respectively, the observations of expected rate of inflation π_t^e, and the unemployment gap ($u_t - u_t^*$).

The GMM estimator of ξ is given by:

$$\hat{\xi} = (X'Z\hat{\Omega}^{-1}Z'X)^{-1}X'Z\hat{\Omega}^{-1}Z'y \tag{3.27}$$

where Z is a T×q matrix of instrumental variables, and $\hat{\Omega}$ is an estimated q×q optimal weighting matrix. The commonly used Newey-West weighting matrix that provides a consistent estimate of Ω (Newey and West, 1987) is applied in this paper. The use of instrumental variables in the GMM approach helps protect against the possibility that the error term ε is correlated with the regressors in equation (3.26), i.e., $E[x_t\varepsilon_t] \neq 0$, which could be caused by omitted variables, or measurement errors embedded in the regressors.

The use of the GMM approach requires the researcher to choose a set of instrumental variables or instruments, for instance, an instrument set of q variables in equation (3.27).

Roberts (2001) pointed out that lagged dependent variables are obvious candidates of instrumental variables in macroeconomic models. In general, the ability of the instruments to capture movements in the variable of interest can be improved by using more lags; however, as cautioned by Roberts (2001), this benefit must be weighted against the danger of using too many instruments in a finite sample. If an instrument set is the same dimension as that of the parameter vector ξ in equation (3.26), then the model is said to be just identified[19]. As the number of instrumental variables q exceeds the number of explanatory variables K, the risk of overfitting and potential estimation bias arise. One diagnostic test to examine the validity of the instrumental variables is the Hansen test of overidentification (Hansen, 1982). Under the null hypothesis that the (q – K) overidentifying restrictions are valid, the Hansen test statistic is defined as:

$$\hat{\varepsilon}' Z \hat{\Omega}^{-1} Z' \hat{\varepsilon} \tag{3.28}$$

has a χ^2_{q-K} limiting distribution, where $\hat{\varepsilon}$ is the estimated residuals from equation (3.26).

To evaluate the estimated single equation estimation, another diagnostic test is to check for potential weakness of the instrument variables. According to Staiger and Stock (1997), the instruments are weak if the partial correlation between the instruments and the included endogenous variable is low. When the instruments are weak, the GMM estimate of ξ in equation (3.26) in general is inconsistent and biased, and inference based on standard asymptotic theory may be unreliable even if the sample size is large. An F-test is suggested by Staiger and Stock (1997) to check for potential weakness of the instrumental variables. Since we are interested in equation (3.26), rewrite it by denoting $X \equiv [X_1, X_2]$ and $\xi \equiv [\xi_1, \xi_2]'$:

$$y = X_1\xi_1 + X_2\xi_2 + \varepsilon \tag{3.29}$$

where X_1 is a $T \times K_1$ matrix of observations on the K_1 endogenous variables, and X_2 is a $T \times K_2$ matrix of exogenous regressors where $K_2 = K - K_1$. The F-test is a test of the null hypothesis of $\Pi = 0$ in the following regression:

$$X_1 = Z\Pi + X_2\Phi + V \tag{3.30}$$

where Z is the same matrix of instrumental variables as in equation (3.27); V is a $T \times 1$ vector of error terms; Π and Φ are respectively a $q \times 1$ vector and a $K_2 \times 1$ vector of parameters to be estimated. If the F-test rejects the null hypothesis, it is said that the empirical evidence is against weakness of instruments, suggesting the instrumental variables used are relevant.

3.2.2. Comparison tests

One motivation of this paper is to compare the empirical performance of the two main alternative specifications of the Phillips curve with U.S. data. To make the comparisons, we follow Paloviita (2002) to apply two statistical tests. The first test is the encompassing test, and the second is the non-nested test (Davidson and MacKinnon, 1993).

The encompassing test, in this paper, is to test whether the currently expected future inflation $E_t[\pi_{t+1}]$, or the previously expected current inflation $E_{t-1}[\pi_t]$ dominates the inflation process π_t. Consider the following model of the inflation rate:

$$\pi_t = \theta E_t[\pi_{t+1}] + (1-\theta)E_{t-1}[\pi_t] + \phi(u_t - u_t^*) \tag{3.31}$$

The sum of the estimated coefficients on $E_t[\pi_{t+1}]$ and $E_{t-1}[\pi_t]$ is restricted to be unity, which allows us to analyze the relative weights of the alternative components in the inflation process. If the null hypothesis of $\theta = 0$ is rejected and the hypothesis of $(1\text{-}\theta) = 0$ is not rejected, then equation (3.31) reduces to the New Keynesian Phillips curve, and the currently expected future inflation is said to dominate the inflation process. If we fail to reject the null hypothesis of $\theta = 0$ but be able to reject the hypothesis of $(1\text{-}\theta) = 0$, then equation (3.31) represents the expectations-augmented Phillips curve, and the previously expected current inflation dominates the inflation process. In other words, the regression equation (3.31) encompasses both specifications of the Phillips curve under consideration as special cases.

The second test, the non-nested test, embeds the two main alternative specifications in a general model, and uses mixing parameters in the combined statistical model. To be more specific, two general models including the alternative specifications of the Phillips curve, as suggested by Palovitta (2002), can be formulated as follows:

$$\pi_t = (1-\alpha)\{\beta E_t[\pi_{t+1}] + \kappa\,(u_t - u_t^*)\} + \alpha\hat{\pi}_t \qquad (3.32)$$

$$\pi_t = (1-\delta)\{\beta E_{t-1}[\pi_t] + \lambda\,(u_t - u_t^*)\} + \delta\tilde{\pi}_t \qquad (3.33)$$

where $\hat{\pi}_t$ is the fitted value obtained from the single equation estimation of the expectations-augmented Phillips curve in equation (3.23), and $\tilde{\pi}_t$ is the fitted value obtained from the single equation estimation of the New Keynesian Phillips curve in equation (3.24). There are four possible conclusions we can make from combining the test results obtained from the above two equations:

(1). If the null hypothesis of $\alpha = 0$ is rejected in equation (3.32), the expectations-augmented Phillips curve is said to have explanatory power over the New Keynesian specification. If the null hypothesis of $\delta = 0$ is not rejected in equation (3.33), the New Keynesian Phillips curve is said to have no explanatory power over the expectations-augmented Phillips curve.

The combined test results provide supporting evidence for the expectations-augmented Phillips curve, and evidence against the New Keynesian Phillips curve.

(2). If we fail to reject the null hypothesis of $\alpha = 0$ in equation (3.32), but at the same time, are able to reject the null hypothesis of $\delta = 0$ in equation (3.33), then we find supporting evidence for the New Keynesian Phillips curve, and evidence against the expectations-augmented Phillips curve.

(3). If both the null hypothesis $\alpha = 0$ in equation (3.32) and the null hypothesis of $\delta = 0$ in equation (3.33) are rejected against the general models, then the conclusion is that neither of the two alternative specifications is satisfactory.

(4). If neither the null hypothesis of $\alpha = 0$ in equation (3.32) nor the null hypothesis of $\delta = 0$ in equation (3.33) is rejected, the combined test results indicate either the data is equally fitted with both specifications, or that the empirical data is poor for testing inflation dynamics with the two alternative specifications of the Phillips curve.

3.3. Empirical Results

3.3.1. Data

The expected rate of inflation is the same time series as that used in the previous Fisher effect analysis, and the sample is from 1982 to 2001 to avoid possible structural breaks. To match the frequency of the expected rate of inflation series, the observations at January, May, and September are selected for the aggregate price and the unemployment rate series. The aggregate price is the monthly consumer price index (CPI) from the International Financial

Statistics, then the inflation rate is defined as the change in the natural log difference between the CPIs, for instance, the inflation rate at May is computed as the log difference between the CPI at May and CPI at January. The observed unemployment rate is the monthly civilian unemployment rate obtained from the U.S. Bureau of Labor Statistics. The unemployment gap is denoted as the difference between the observed unemployment rate and the NAIRU. Since the NAIRU is an unobservable quantity, we need an estimate of the NAIRU to construct the unemployment gap series. The software WINRATS is used in this study.

In general, there are two frequently used approaches to measure the NAIRU. The first approach assumes that the NAIRU is constant, and then the NAIRU is estimated via the regression intercept over the sample period (Gordon, 1994; Fuhrer, 1995; and Tootell, 1994). The assumption of a constant NAIRU has been questioned by many economists, and there is a growing literature that seeks to estimate the path of a time varying NAIRU. With the emphasis on the time variability of the NAIRU, the second approach extracts a smoothed or trend series from the observed unemployment rate series via the use of statistical smoothing or de-trending algorithms, one of which is the H-P filter (Hodrick and Prescott, 1997)[20]. This approach has been used in empirical studies such as Ball and Mankiw (2002), and Ferreira, Aguirre, and Gomes (2003).

In this study, the H-P filter technique is used to derive estimates of the NAIRU. The choice of a smoothing parameter required to implement the H-P filter is largely arbitrary, and we will use two alternative H-P smoothing parameters. The first smoothing parameter is 1600, suggested by Hodrick and Prescott (1997), which is a standard choice for the H-P smoothing parameter, especially for quarterly data. The second smoothing parameter is 506.25, calculated according to the formula recommended by Ravn and Uhlig (2001)[21].

Based on the estimated NAIRU using the smoothing parameters 1600 and 506.25, we obtain two unemployment gap series, referred to as UGAP1 and UGAP2 in the remainder of this study. We have shown earlier in this paper that the expected rate of inflation series is stationary. To avoid possible spurious regression, the stationarities of the inflation rate and the two unemployment gap series are examined by the ADF test and the Phillips-Perron test.

Table 3.2 reports the unit root test results. The lag length in the ADF tests is selected to be two for all three series because the coefficient estimates on higher orders of the lags are not significantly different from zero. The critical value at the 5 percent level is -2.89 for the ADF test with the constant, and -1.95 for the ADF test without the constant. In both cases, we are able to reject the null hypothesis of a unit root at the 5 percent level for the inflation rate, and the two unemployment gap series. The Phillips-Perron test results tell a similar story. Given the critical value -2.89 at the 5 percent significance level for the Phillips-Perron test, we can reject the null hypothesis of the presence of a unit root in the inflation rate and the unemployment gap series UGAP2, but we cannot reject the null hypothesis that the unemployment gap series UGAP1 contains a unit root at the 5 percent level. However, at the 10 percent significance level, we are able to reject the null of the presence of a unit root, and conclude that the unemployment gap series UGAP1 is stationary. Therefore, we proceed under the assumption that all the variables used in this study are stationary.

3.3.2. Estimation of the single equation

Using the GMM approach, we estimate separately the expectations-augmented Phillips curve specified in equation (3.23), and the New Keynesian Phillips curve specified in equation (3.24). For comparative purpose, we follow Paloviita (2002) and use the same instrument set for both specifications. Our instrument set includes three variables: the lagged expected rate of inflation, and two lags of the unemployment gap. Two alternative unemployment gap series, the UGAP1 and the UGAP2, are employed in the estimation. Table 3.3 reports the estimates of both models using a 13-lag Newey-West weighting matrix[22].

The estimation results are quite plausible over the sample period. For both specifications of the Phillips curve using the UGAP1 as the measure of unemployment gap, the estimated coefficients on expected inflation are positive and significant, which is consistent with the underlying theory. In particular, the point estimates on expected inflation are about 0.96, and we cannot reject the null of $\beta = 1$ at the 5 percent significance level. The coefficients associated with the unemployment gap have the expected negative sign, though the estimated coefficients are not significantly different from zero at the 5 percent level in either the expectations-augmented Phillips curve or the New Keynesian Phillips curve. One possible reason for the insignificant estimated coefficients on the unemployment gap is that the expected inflation has already incorporated information contained in the unemployment gap. Similar coefficient estimates (shown in Table 3.3) and the same conclusions are obtained when the UGAP2 is employed as the measure of unemployment gap in the estimation of equations (3.23) and (3.24).

Two diagnostic tests are conducted to evaluate the estimation results reported in Table 3.3: Hansen's test of validity of the overidentifying restrictions, and an F-test checking potential weakness of the instrumental variables. The chi-square statistics of the Hansen test are shown in the last column of Table 3.3. We fail to reject the null hypothesis and conclude that the overidentifying restrictions are valid for both specifications of the Phillips curve with either unemployment gap series, the UGAP1 or the UGAP2. In addition to the Hansen test, potential weakness of the instrumental variables is checked by an F-test of $\Pi = 0$ in equation (3.30), where the matrix of X_1 represents the unemployment gap, and X_2 represents the expected rate of inflation[23]. The calculated F-statistic is 151.81 with a p-value = 0.00, and 99.84 with a p-value = 0.00 when the UGAP1 and UGAP2 are respectively used as the measure of unemployment gap in the regression described in equation (3.30). In both cases, the null hypothesis of $\Pi = 0$ is strongly rejected at the 5 percent significance level suggesting the instrumental variables seem to be relevant. Taken together, the results of the two diagnostic tests indicate that both specifications of the Phillips curve work well and the inferences based on empirical models are reliable.

Summing up this subsection, both the expectations-augmented Phillips curve and the New Keynesian Phillips curve seem to capture inflation dynamics fairly well when we use the constructed expected rate of inflation as a proxy for inflation expectations. The estimated coefficients on the expected rate of inflation were positive and significant in both specifications, and the estimated coefficients on the unemployment gap had the expected negative sign. However, the statistical preference for either specification of the Phillips curve cannot be claimed from the single equation estimation results. The test results presented in the next subsection may be helpful in this regard.

3.3.3. Comparison test results

Two statistical tests, the encompassing test and the non-nested test, are conducted to compare the two main alternative specifications of the unemployment gap based Phillips curve. To allow for comparison of results from the two statistical tests, we use the same instrument set in both tests: the three instrumental variables used in the single equation estimations, and an additional lag of the expected rate of inflation[24]. Moreover, the Hansen test will be applied to check whether the overidentifying restrictions are valid.

The encompassing test described in equation (3.31) tests whether the currently expected future inflation, $E_t[\pi_{t+1}]$, or the previously expected current inflation, $E_{t-1}[\pi_t]$, dominates the inflation process. When the UGAP1 is employed as the measure of the unemployment gap in the encompassing test, the parameter estimates and the standard errors shown in parentheses are:

$$\underset{}{\pi_t} = \underset{(0.385)}{0.691}E_t[\pi_{t+1}] + \underset{(0.385)}{0.309}E_{t-1}[\pi_t] - \underset{(0.107)}{0.106}(u_t - u_t^*) \tag{3.34}$$

The point estimate on the currently expected future inflation is about 0.69 and is significant at the 10 percent level, while the point estimate on the previously expected current inflation is relatively small and not statistically different from zero. Therefore, the currently expected future inflation appears to dominate the inflation process compared with the previously expected current inflation. The estimated coefficient on the unemployment gap has the expected negative sign, but it is not statistically different from zero at the 5 percent significance level. The chi-square statistic of the Hansen test is 0.93 with a p-value = 0.34, so we fail to reject the null hypothesis that the overidentifying restrictions are valid at conventional significance levels. When the UGAP2 is used as the measure of unemployment gap in the encompassing test, the results are similar to that when the UGAP1 is used in the test:

$$\pi_t = \underset{(0.384)}{0.735}E_t[\pi_{t+1}] + \underset{(0.384)}{0.265}E_{t-1}[\pi_t] - \underset{(0.145)}{0.138}(u_t - u_t^*) \tag{3.35}$$

The diagnostic check indicates that the overidentifying restrictions are valid at conventional significance levels since the chi-square statistic of the Hansen test is 1.02 with a p-value = 0.31. Overall, the encompassing test appears to support the New Keynesian specification of the Phillips curve.

In addition to the encompassing test, the two specifications of the Phillips curve are compared using the non-nested test by estimating equations (3.32) and (3.33). Table 3.4 provides the non-nested test results including the parameter estimates, the associated standard errors, and the chi-square statistics from the Hansen test. When the UGAP1 is used as the measure of unemployment gap in the non-nested test, we can not reject the null hypothesis of $\alpha = 0$ in equation (3.32) at conventional significance levels, suggesting that the expectations-augmented Phillips curve has no explanatory power over the New Keynesian Phillips curve. At the same time, we are able to reject the null hypothesis of $\delta = 0$ in equation (3.33) at the 1 percent significance level, indicating that the New Keynesian Phillips curve has explanatory power over the expectations-augmented Phillips curve. Combining the above test results from equations (3.32) and (3.33), we conclude that the New Keynesian Phillips curve is preferred over the expectations-augmented Phillips curve with the U.S. data. The same conclusion can be obtained in the non-nested test when the UGAP2 is used as the measure of the unemployment gap in the estimation of equations (3.32) and (3.33).

All in all, the encompassing and the non-nested tests results provide evidence in favor of the New Keynesian Phillips curve even though the expectations-augmented Phillips curve provided a reasonable description of inflation dynamics with the U.S. data. This finding is in line with the previous study by Palovitta (2002) who showed supporting evidence for the New Keynesian Phillips curve with the Euro data using the encompassing and the non-nested tests.

The above conclusions base on the estimation using the constructed expected rate of inflation incorporating information in the commodity futures market as a measure of inflation expectations. To compare the empirical superiority of the New Keynesian Phillips curve and the expectations-augmented Phillips curve, we employ another proxy for inflation expectations derived from an ARMA(3,0) time series model in the previous study. The comparison tests' results are reported in Table 3.5.

The encompassing test results shown in Panel A of Table 3.5 indicate that the currently expected future inflation appears to dominate the inflation process compared with the previously expected current inflation when either the UGAP1 or the UGAP2 is employed as the measure of the unemployment gap, hence the encompassing test appears to support the New Keynesian specification of the Phillips curve. The non-nested test results shown in Panel B of Table 3.5 also suggest that the New Keynesian Phillips curve is preferred over the expectations-augmented Phillips curve regardless of the use of alternative measures of the unemployment gap[25]. Therefore, the encompassing and the non-nested tests results provide evidence in favor of the New Keynesian Phillips curve with the U.S. data when the proxy for inflation expectations is derived from a time series model. This finding is consistent with the previous comparison tests' results when the constructed expected rate of inflation from Hamilton's procedure is used as the proxy for inflation expectations.

4. Summary

In this paper, we have tried to provide some insight into the empirical testing of the Fisher effect and the Phillips curve by employing an alternative measure of inflation expectations for the United States over the period 1982-2001. The constructed expected rate of inflation series is derived by incorporating information from commodity futures prices, which are considered to respond quickly to new information available in the commodity futures market, including changes in monetary policy and expected inflation. Such a measure of inflation expectations is believed to track any expected change in inflation more closely than conventional time series or survey methods and hence is a more accurate measure of people's true expectations. Based on the estimation using this alternative measure of inflation expectations, the empirical testing results of the Fisher effect hypothesis and the Phillips curve may be more reliable than (or at lease reliable as) those using other measures of inflation expectations.

Tests of the Fisher hypothesis have attracted lots of attention, and the results in general support the existence of the long-run Fisher effect, but the short-run Fisher effect has limited empirical support. This paper focuses on the reexamination of the short-run Fisher effect. The FGLS regression results show the estimated coefficient for the interest rate term is positive and significant, which provides support for the existence of the short-run Fisher effect. Such findings suggest a need for caution in using the nominal interest rate as an indicator of the stance of monetary policy since changes in the nominal interest rate reflect changes in the expected inflation rate rather than the real interest rate.

Since the early 1990s, the coexistence of low unemployment and low inflation in the United States has caused renewed interest in studying the Phillips curve. This study is the first attempt to compare the empirical superiority of the two main alternative specifications of the unemployment gap-based Phillips curve, the expectations-augmented Phillips curve, and the New Keynesian Phillips curve, using the encompassing and non-nested tests for the

United States. Although single equation estimation by the GMM approach suggests that both specifications of the Phillips curve seem to capture inflation dynamics quite reasonably, the comparison tests results present evidence in favor of the New Keynesian Phillips curve, which is useful to monetary policymakers as a guide to policy change. Under the New Keynesian Phillips curve, monetary policy changes inflation immediately; the better the policy is understood by economic agents, the more current inflation is affected.

The short-run Fisher effect and the superiority of the two main alternative specifications of the Phillips curve are also examined using a proxy derived from a time series model. We find evidence against the presence of the short-run Fisher effect, but evidence in favor of the New Keynesian Phillips curve when using this proxy. Compared with the results using the constructed expected rate of inflation incorporating information from the commodity futures market, the empirical finding is consistent with that in the comparison of the two main alternative specifications of the Phillips curve. However, the finding is inconsistent with that in the examination of the short-run Fisher effect, and we believe the reason for this difference may come from the fact that the proxy derived from the time series model is a less accurate measure of inflation expectations.

Future attempts can be made to examine the Fisher relationship in the generalized form that takes into account the tax effect on the nominal return, or to examine and compare the two main specifications of the Phillips curve by using alternative indicators of excess demand. The long-run equilibrium relationship between the expected inflation and the interest rate, or the long-run inflation-unemployment tradeoff can be investigated if a longer sample period is available. It also would be interesting to explore a nonlinear Phillips curve or a nonlinear relationship between the expected inflation and the interest rate. Moreover, future research work is desirable in the development of new methodology that focuses on deriving alternative, preferable more accurate measure of inflation expectations, with which can provide further evidence in the investigation of some key economic theories in which a key role is played by inflation expectations.

Notes

1. Roberts (1995) examined the New Keynesian Phillips curve specified as $\Delta p_t - E_t \Delta p_{t+1} = \alpha + \beta y_t + \varepsilon_t$, where y_t represents the excess demand. He employed the actual future value of inflation as a proxy for the expectation, then the model is estimated in the form: $\Delta p_t - \Delta p_{t+1} = \alpha + \beta y_t + \varepsilon_t + v_t$, where $v_t = E_t \Delta p_{t+1} - \Delta p_{t+1}$ and is an additional source of error.

2. Olekalns (1996) stated that the strong form of the Fisher hypothesis required assumptions regarding the absence of taxation, and a zero interest elasticity of money demand. Once relaxing these assumptions, the weaker form of the Fisher hypothesis is more realistic. For instance, an interest-elastic money demand function implies partial adjustment, while taxation implies that the nominal interest rate should include a premium over and above the expected inflation.

3. The Box-Jenkins methodology is a three-stage method (including the identification stage, the estimation stage and diagnostic checking stage) that can be used to select an appropriate model for estimating and forecasting a univariate time series.

4. Alternatively, the test for the long-run Fisher effect under the assumption of rational expectations was to test for a unit root in the ex ante real interest rate rr_t^m defined as $rr_t^m = i_t^m - E_t[\pi_t^m]$.

5. The inflation and interest rates move like independent random walks within the band, but as the inflation breaches the threshold, the policy maker will actively pursue monetary policies that aim to bring inflation back inside the band whenever the inflation rate falls outside such band of tolerable inflation.

6. The plot of the expected inflation series appears to show seasonality. By regressing the expected rate of inflation on two seasonal dummies (May dummy & September dummy), we find the coefficient on the May dummy is significant at the 10 percent level, and the estimated intercept representing the January dummy is significant at the 5 percent level. Therefore, seasonality exists but it is uncertain what causes this problem.

7. The Phillips-Perron test statistics indicate that we cannot reject the null of the presence of a unit root at the 5 percent level, but we can reject the null at the 10 percent level for the expected rate of inflation when the lag lengths are one and three. However, Ng and Perron (2001) pointed out that the inflation rate is a time series that often exhibits a large negative moving-average root, in which case, many unit root tests display significant size distortion resulting in over rejection of

the unit root null hypothesis, but their proposed Ng-Perron test has improved size and power. The calculated Ng-Perron test statistic is -0.88, compared with the critical value -1.98 at the 5 percent significance level, we cannot reject the presence of a unit root in the expected rate of inflation series.

8. The calculated Ljung-Box Q-statistics at lag 3, 6, and 12 are respectively 11.17 (p-value=0.01), 23.79 (p-value=0.00), and 31.73 (p-value=0.00). The Ljung-Box Q-statistic is specified as $Q = T(T+2)\sum_{k=1}^{s} r_k^2 /(T-k) \sim \chi_s^2$, where T is the number of observations, and r_k is the sample autocorrelation function. If the calculated Q exceeds the critical value, then at least one value of r_k is statistically different from zero at the specified significance level. It can be used as a check to see if the residuals from an estimated ARMA(p,q) model are a white-noise process, and Q has a chi-square distribution with s-p-q degrees of freedom.

9. The calculated Ljung-Box Q-statistics at lag 3, 6, and 12 are respectively 2.10 (p-value=0.55), 12.10 (p-value=0.06), and 19.15 (p-value=0.09).

10. The parameter estimate of β in equation (3.12) is about 0.424, but the Ljung-Box Q-statistics indicate the presence of serial correlation in the regression errors, which seems to be adequately represented by an AR(5) model. To correct for the serial correlation, equation (3.12) is re-estimated by the FGLS estimator, and results are shown in equation (3.20).

11. Friedman (1968) developed the natural rate hypothesis and drew the distinction between the short-run and long-run Phillips curve trade-off: there is always a temporary trade-off between inflation and unemployment, but there is no permanent trade-off based on the natural rate theory. The natural rate theory states the Phillips curve tradeoff is vertical at the natural rate of unemployment in the long run. The natural rate of unemployment is the rate at which there is no upward or downward pressure on wage rates, or the rate that prevails when expectations are fully realized and incorporated into wages and prices. In this study, the Phillips curve refers to the short-run relation, and our literature review focuses on studies for the United States.

12. The NAIRU is the unemployment rate that is consistent with stable inflation, below which inflation tends to rise and above which inflation tends to fall. The NAIRU is interchangeable with the natural rate of unemployment in the linear specification of the Phillips curves.

13. Additional variables such as oil price, changes in the relative price of imports, change of relative price of food and energy, and dummy variables have also been incorporated to capture various supply shocks in the empirical studies of the Phillips curve.

14. The adaptive expected rate of inflation was the actual inflation rate in the previous period. The rational inflation expectation was derived from regressing inflation rate on past inflation, a time trend, real growth rate of money supply, and real GDP.

15. The restrained behavior of prices since the end of the 1990-1991 recession to mid 1990s is referred to as an inflation puzzle by economists. The coexistence of low unemployment and low inflation has caused renewed academic attention on the Phillips curve relation and led empirical studies into different directions, a couple of which are listed as follows. Different measures of excess demand instead of the unemployment gap have been used, for example, marginal cost is used as a measure of excess demand in the Phillips curve studies by Gali and Gertler (1999), and Gali, Gertler, and Lopez-Salido (2001). The instability of the Phillips curve

for other countries are explored, for example, in Kichian's (2001) work with the Canadian data, and Ferreira, Aguirre, and Gomes's (2003) study with the Brazilian data. Alternative Phillips curve models are also proposed, such as the "triangle" model of Gordon (1997), and the sticky-information Phillips curve of Mankiw and Reis (2001).

16. The behavior of compensation growth is consistent with that for the benefits and wages representing total labor cost. The compensation growth variable in Lown and Rich's (1997) study is the growth rate of compensation per hour for the non-farm business sector reported by the Department of Labor, Bureau of Labor Statistics. Compensation comprises wages and salaries for workers plus employers' contributions for social security insurance and private benefit plans; it also includes an estimate of wages, salaries, and supplemental payments for self-employed workers.

17. The other category of the sticky price models in the New Keynesian literature is the state-dependent model (Ball, Mankiw & Romer, 1988). The state-dependent model says that firms will change prices when underlying determinants such as demand or costs reaches certain bounds. In general, the state-dependent models have no explicit closed-form solutions but the time-dependent models have. Consequently, the time-dependent models are more popular.

18. The coefficient κ is a function of β and θ, specified as $\kappa=(1-\theta)(1-\beta\theta)/\theta$. In the framework where θ represents the degree of price rigidity, κ is decreasing in θ, the longer prices are fixed on average, the less sensitive inflation is to current variation in excess demand. As a result, forward-looking firms have to set prices for possible multiple periods and base their pricing decisions on the expected future developments of excess demand.

19. Consider a linear regression model $y_t = x_t'\xi + \varepsilon_t$, where x_t is a T×K matrix of observations. Suppose it is known that $E[x_t\varepsilon_t] = 0$, and $E[x_t(y_t-x_t'\xi)] \neq 0$ when $\xi \neq \xi_0$. In such a case, there are K moment conditions and K unknown parameters, this model is said to be exactly identified, and OLS estimates are consistent and unbiased. Suppose now that $E[x_t\varepsilon_t] \neq 0$, and a set of instrumental variables Z with dimension $q > K$ is used in the GMM regression based on the assumption that $E[z_t\varepsilon_t] = 0$ which is referred to as the orthogonality condition. In this case, the number of orthogonality conditions q is greater than the number of parameters K to be estimated, this model is said to be overidentified.

20. The H-P filter is a generalization of a linear time trend, which allows the slope of the trend to change gradually over time. It minimizes the sum of squared deviations between the trend and the actual series with a penalty for curvature that keeps the trend smooth. The H-P filter yields a linear time trend if the penalty is very high, while the filter yields the original series if there is no penalty.

21. Ravn and Uhlig (2001) suggested that the smoothing parameter in the H-P filter should move with the fourth power of the frequency of observations according to the following formula: $\lambda_s = s^4\lambda_1$, where λ_1 is the standard choice of 1600 for quarterly data, and s is the ratio of the frequency of observation compared to quarterly data. In our study, the sample is tri-annual data, so s = ¾, and λ_s = 506.25.

22. The choice of 13 lags in the Newey-West weighting matrix is based on the following evidence: the autocorrelation coefficients for the residual series of the instrumental regression before

applying the weighting matrix in the New Keynesian Phillips curve are significant at lags 11, 12 and 13. At lag 13, the estimated ACF is -0.31, comparing with the critical value -0.26 at the 5 percent significance level, we reject the null of zero autocorrelation coefficients, and conclude that the autocorrelation still exists at lag 13. The lag length 13 in the weighting matrix is also large enough to get rid of serial correlations in the residual series obtained in other model estimations, which provide similar coefficient estimates when using different lags.

23. The expected rate of inflation is a predetermined variable, so we treat its lagged term as an exogenous variable, then the unemployment gap is the only right-hand side endogenous variable in equation (3.29).

24. The use of the instrument set with three instruments in the single equation estimations provides insignificant estimates due to large standard errors, which may be caused by omitted variables. The inclusion of an additional instrument provides plausible estimates for the comparison tests.

25. The 13-lag Newey-West weighting matrix is used, which is the same as in the comparison tests when the constructed expected rate of inflation is employed as the proxy for inflation expectation. Results from the empirical tests using different lag lengths of the Newey-West weighting matrix are similar. The diagnostic check indicates that the overidentifying restrictions are valid at conventional significance levels. Note the single equation estimation results are similar to those when the proxy for inflation expectations is derived from Hamilton's procedure. Both seem to capture inflation dynamics fairly well: the estimated coefficients on expected inflation are positive and significant which is consistent with the underlying theory; the coefficients on the unemployment gap have the expected negative sign.

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Table 3.1. The unit root test of expected rate of inflation and interest rate

Variable y_t	Hypothesis	1975:II-2001:III		1982:I-2001:III	
		Lags	Test statistic	Lags	Test statistic
Panel A. Augmented Dickey Fuller Test					
	$\Delta y_t = \alpha + \rho y_{t-1} + \beta t + \sum_{i=2}^{p} \phi_i \Delta y_{t-i+1} + \varepsilon_t$				
Expected rate of inflation	H_0: $\rho=0$	3	-2.75	2	-5.08**
	H_0: $\alpha=\rho=\beta=0$		2.62		7.16**
	H_0: $\rho=\beta=0$		3.86		9.99**
3-month Treasury bill rate	H_0: $\rho=0$	1	-2.54	1	-3.93**
	H_0: $\alpha=\rho=\beta=0$		2.33		4.90**
	H_0: $\rho=\beta=0$		3.44		6.58**
	$\Delta y_t = \alpha + \rho y_{t-1} + \sum_{i=2}^{p} \phi_i \Delta y_{t-i+1} + \varepsilon_t$				
Expected rate of inflation	H_0: $\rho=0$	3	-1.50	2	-5.11**
	H_0: $\alpha=\rho=0$		1.19		9.82**
3-month Treasury bill rate	H_0: $\rho=0$	1	-1.62	1	-3.40**
	H_0: $\alpha=\rho=0$		1.37		5.68**
	$\Delta y_t = \rho y_{t-1} + \sum_{i=2}^{p} \phi_i \Delta y_{t-i+1} + \varepsilon_t$				
Expected rate of inflation	H_0: $\rho=0$	3	-1.06	2	-3.04**
3-month Treasury bill rate	H_0: $\rho=0$	1	-0.74	1	-2.43**
Panel B. Phillips-Perron Test					
Expected rate of inflation	H_0: $\rho=0$	1	-2.64*	1	-5.40**
	H_0: $\rho=0$	2	-2.49	2	-5.45**
	H_0: $\rho=0$	3	-2.72*	3	-5.66**
3-month Treasury bill rate	H_0: $\rho=0$	1	-1.87	1	-2.72*
	H_0: $\rho=0$	2	-1.88	2	-2.74*
	H_0: $\rho=0$	3	-1.91	3	-2.75*

Note: 1). *, ** indicate significance at the 10% and 5% level, respectively.

2). Ljung-Box Q-statistics show that the residuals from the fitted model appear to be white noise at the 5% level.

3). At the 10%, and 5% significance level, the critical values for the ADF test with the constant and trend are respectively -3.18, and -3.45; for the ADF test with only the constant are respectively -2.58, and -2.89; and for the ADF test without the constant and trend are respectively -1.61, and -1.95.

4). At the 10%, and 5% significance level, the critical values for the F-statistics ϕ_1 are respectively 3.86, and 4.71; for ϕ_2 are respectively 4.16, and 4.88; and for ϕ_3 are respectively 5.47, and 6.49.

5). At the 10%, and 5% significance level, the critical values for the Phillips-Perron test are respectively -2.58, and -2.89.

Table 3.2. The unit root tests of the inflation rate and unemployment gaps over the period 1982-2001

Unemployment gap	Lags	Hypothesis	Test statistic
Panel A. Augmented Dickey Fuller Test			
	$\Delta y_t = \alpha + \rho\, y_{t-1} + \sum_{i=2}^{p} \phi_i \Delta y_{t-i+1} + \varepsilon_t$		
Inflation rate	2	H_0: ρ=0	-5.59**
UGAP1	2	H_0: ρ=0	-3.69**
UGAP2	2	H_0: ρ=0	-4.31**
	$\Delta y_t = \rho\, y_{t-1} + \sum_{i=2}^{p} \phi_i \Delta y_{t-i+1} + \varepsilon_t$		
Inflation rate	2	H_0: ρ=0	-5.65**
UGAP1	2	H_0: ρ=0	-3.72**
UGAP2	2	H_0: ρ=0	-4.35**
Panel B. Phillips-Perron Test			
Inflation rate	2	H_0: ρ=0	-7.43**
UGAP1	2	H_0: ρ=0	-2.88*
UGAP2	2	H_0: ρ=0	-3.62**

Note: *, ** indicate significance at the 10% and 5% level, respectively.

Table 3.3. The single equation estimation results

Panel A. Expectation-augmented Phillips curve: $\pi_t = \beta E_{t-1}[\pi_t] + \lambda (u_t - u_t^*) + \varepsilon_t$

	β (std. error)	λ (std. error)	χ_1^2 (p-value)
UGAP1	0.961** (0.018)	-0.082 (0.052)	1.381 (0.240)
UGAP2	0.962** (0.018)	-0.078 (0.075)	1.393 (0.238)

Panel B. New-Keynesian Phillips curve: $\pi_t = \beta E_t[\pi_{t+1}] + \kappa (u_t - u_t^*) + \varepsilon_t$

	β (std. error)	κ (std. error)	χ_1^2 (p-value)
UGAP1	0.963** (0.032)	-0.106 (0.132)	0.876 (0.349)
UGAP2	0.966** (0.031)	-0.149 (0.182)	0.951 (0.329)

Note: *, ** indicate significance at the 5% and 1% level, respectively.

Table 3.4. The non-nested test results for the two alternative specifications of the Phillips curve

Panel A. non-nested test: $\pi_t = (1-\alpha)\{\beta E_t[\pi_{t+1}] + \kappa\ (u_t - u_t^*)\} + \alpha\hat{\pi}_t$

	α (std. error)	β (std. error)	κ (std. error)	χ_1^2 (p-value)
UGAP1	0.289	0.965*	-0.109	0.929
	(0.400)	(0.541)	(0.153)	(0.335)
UGAP2	0.305	0.968*	-0.148	1.020
	(0.400)	(0.557)	(0.214)	(0.312)

Panel B. non-nested test: $\pi_t = (1-\delta)\{\beta E_{t-1}[\pi_t] + \lambda\ (u_t - u_t^*)\} + \delta\tilde{\pi}_t$

	δ (std. error)	β (std. error)	λ (std. error)	χ_1^2 (p-value)
UGAP1	0.604*	0.703	-0.178	0.929
	(0.339)	(0.973)	(0.277)	(0.335)
UGAP2	0.591*	0.719	-0.215	1.020
	(0.340)	(0.941)	(0.358)	(0.312)

Note: *, ** indicate significance at the 10% and 5% level, respectively.

Table 3.5. The comparison test results using expected inflation rate derived from the time series model

Panel A. Encompassing test

$$\pi_t = \theta E_t[\pi_{t+1}] + (1-\theta) E_{t-1}[\pi_t] + \phi(u_t - u_t^*)$$

	θ (std. error)	1-θ (std. error)	ϕ (std. error)	χ_1^2 (p-value)
UGAP1	1.712**	-0.712**	-0.012	0.126
	(0.342)	(0.342)	(0.150)	(0.723)
UGAP2	1.712**	-0.712**	0.022	0.225
	(0.357)	(0.357)	(0.187)	(0.635)

Panel B. Non-nested test

(1). $\pi_t = (1-\alpha)\{\beta E_t[\pi_{t+1}] + \kappa\ (u_t - u_t^*)\} + \alpha\hat{\pi}_t$

	α (std. error)	β (std. error)	κ (std. error)	χ_1^2 (p-value)
UGAP1	-0.210	0.935*	-0.146	0.126
	(0.506)	(0.398)	(0.169)	(0.723)
UGAP2	-0.221	0.936*	-0.116	0.225
	(0.529)	(0.411)	(0.207)	(0.635)

(2). $\pi_t = (1-\delta)\{\beta E_{t-1}[\pi_t] + \lambda\ (u_t - u_t^*)\} + \delta\tilde{\pi}_t$

	δ (std. error)	β (std. error)	λ (std. error)	χ_1^2 (p-value)
UGAP1	1.235**	0.809	-0.357	0.126
	(0.525)	(1.951)	(0.849)	(0.723)
UGAP2	1.247**	0.810	-0.297	0.225
	(0.548)	(1.935)	(0.863)	(0.635)

Note: *, ** indicate significance at the 10% and 5% level, respectively.

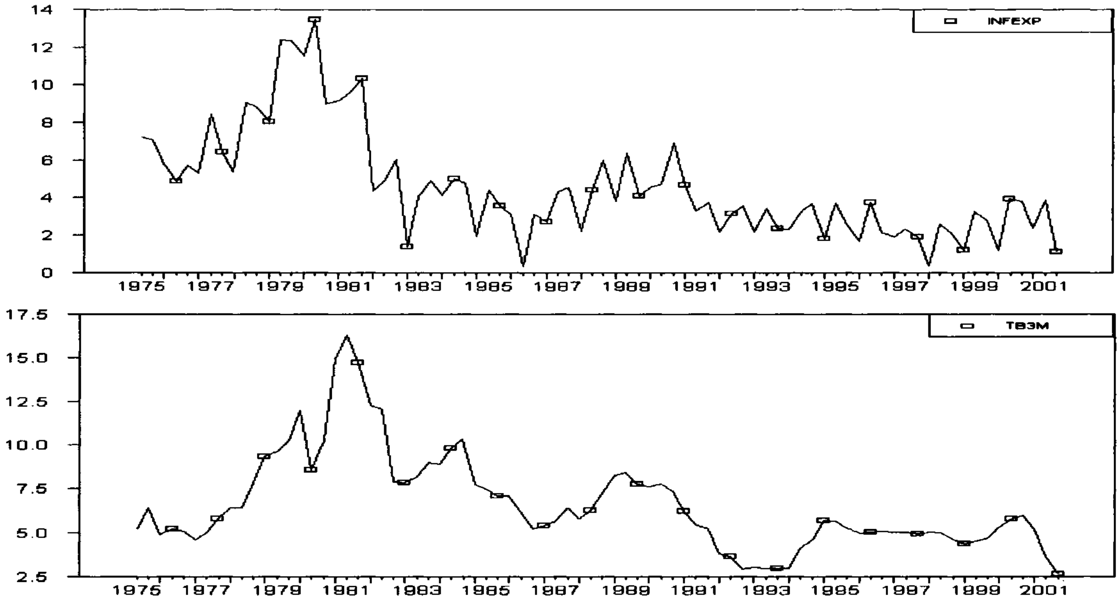

Figure 3.1. The expected inflation and the interest rates series over 1975:II-2001:III

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